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Methods for detrending success metrics to account for inflationary and deflationary factors*

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Abstract. Time-dependent economic, technological, and social factors can artificially inflate or deflate quantitative measures for career success. Here we develop and test a statistical method for normalizing career success metrics across time dependent factors. In particular, this method addresses the long standing question: how do we compare the career achievements of professional athletes from different historical eras? Developing an objective approach will be of particular importance over the next decade as *major league baseball* (MLB) players from the "steroids era" become eligible for Hall of Fame induction. Some experts are calling for *asterisks* (*) to be placed next to the career statistics of athletes found guilty of using performance enhancing drugs (PED). Here we address this issue, as well as the general problem of comparing statistics from distinct eras, by detrending the seasonal statistics of professional baseball players. We detrend player statistics by normalizing achievements to seasonal averages, which accounts for changes in relative player ability resulting from a range of factors. Our methods are general, and can be extended to various arenas of competition where time-dependent factors play a key role. For five statistical categories, we compare the probability density function (pdf) of detrended career statistics to the pdf of raw career statistics calculated for all player careers in the 90-year period 1920–2009. We find that the functional form of these pdfs is stationary under detrending. This stationarity implies that the statistical regularity observed in the right-skewed distributions for longevity and success in professional sports arises from both the wide range of intrinsic talent among athletes and the underlying nature of competition. We fit the pdfs for career success by the Gamma distribution in order to calculate objective benchmarks based on extreme statistics which can be used for the identification of extraordinary careers.

1 Introduction

Quantitative measures for success are important for comparing both individual and group accomplishments [1], often achieved in different time periods. However, the evolutionary nature of competition results in a non-stationary rate of success, that makes comparing accomplishments across time statistically biased. The analysis of sports records reveals that the interplay between technology and ecophysiological limits results in a complex rate of record progression [2–4]. Since record events correspond to extreme achievements, a natural follow-up question is: how does the success rate of more common achievements evolve in competitive arenas? To answer this question, we analyze the evolution of success, and the resulting implications on metrics for career success, for all Major League Baseball (MLB) players over the entire history of the game. We use concepts from statistical physics to identify statistical regularity in success, ranging from common to extraordinary careers.

1.1 Baseball

The game of baseball has a rich history, full of scandal, drama and controversy [5]. Indeed, the importance of baseball in American culture is evident in the game's longevity, having survived the Great Depression, two World Wars, racial integration, free agency, and multiple player strikes. When comparing players from different time periods it is often necessary to rely purely on statistics, due to the simple fact that Major League Baseball's 130+ year history spans so many human generations, extending back to a time period before television and even before public radio.

Luckily, due to the invention of the box score very early in the evolution of the game, baseball has an extremely rich statistical history. When comparing two players, objectively determining who is better should be as

* Tables S1–S10 are only available in electronic form at www.epj.org

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straightforward as comparing their statistics. However, the results of such a naive approach can be unsatisfying. This is due to the fact that the history of professional baseball is typically thought of as a collection of ill-defined, often overlapping eras, such as the "deadball" era, the "liveball" era, and recently, the "steroids" era of the 1990's and 2000's. As a result, many careers span at least two such eras.

The use of statistics, while invaluable to any discussion or argument, requires proper contextual interpretation. This is especially relevant when dealing with the comparison of baseball careers from significantly different periods. Among common fans, there will always be arguments and intergenerational debates. Closely related to these debates, but on a grander stage, is the election process for elite baseball players into the great bastion of baseball history, the National Baseball Hall of Fame (HOF). In particular, an unbiased method for quantifying career achievement would be extremely useful in addressing two issues which are on the horizon for the HOF:

(i) How should the HOF reform the election procedures of the veterans committee, which is a special committee responsible for the retroactive induction of players who were initially overlooked during their tenure on the HOF ballot. Retroactive induction is the only way a player can be inducted into the HOF once their voting tally drops below a 5% threshold, after which they are not considered on future ballots. Closely related to induction through the veterans committee is the induction of deserving African American players who were not allowed to compete in MLB prior to 1947, but who excelled in the Negro Leagues, a separate baseball league established for "players of color". In 2006, the HOF welcomed seventeen Negro Leaguers in a special induction to the HOF.

(ii) How should the HOF deal with players from the "steroid" era (1990's–2000's) when they become eligible for HOF induction. The *Mitchell Report* [6] revealed that more than 5% of players in 2003 were using PED. Hence, is right to celebrate the accomplishments of players guilty of using PED more than the accomplishments of the players who were almost as good and were not guilty of using PED? Similarly, how can we fairly assess player accomplishments from the steroids era without discounting the accomplishments of innocent players?

Here we address the era dependence of player statistics in a straightforward way. We develop a quantitative method to "detrend" seasonal statistics by the corresponding league-wide average. As a result, we normalize accomplishments across *all* possible performance factors inherent to a given time period. Our results provide an unbiased and statistically robust appraisal of career achievement, which can be extended to other sports and other professions where metrics for success are available.

This paper is organized as follows: in Section 1.2 we first analyze the distribution of career longevity and success for all players in Sean Lahman's Baseball Archive [7], which has player data for the 139-year period 1871–2009.

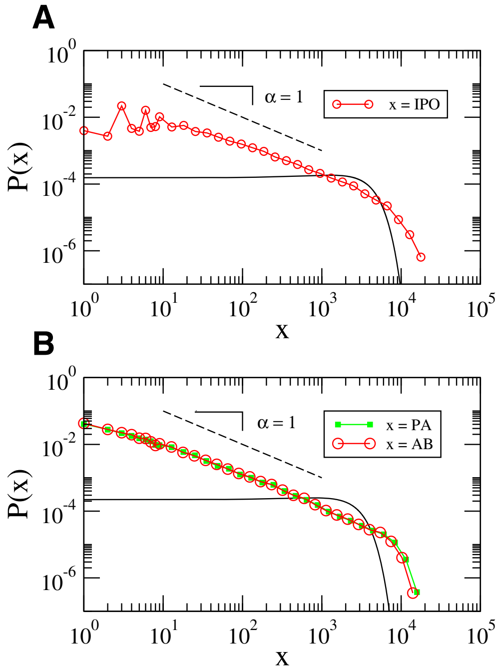

Fig. 1. The probability density functions (pdfs) of player longevity demonstrate the wide range of careers. We define longevity as (A) the number of outs-pitched (IPO) for pitchers and (B) the number of at-bats (AB) for batters. (B) For comparison, we also plot the pdf for plate appearances (PA) for batters, which is not significantly different than the pdf for AB. Calculated from all careers ending in the 90-year period 1920–2009, these extremely right-skewed distributions for career longevity extend over more than three orders of magnitude. For comparison of the scaling regime, we plot a power-law with exponent $\alpha = 1$ (dashed black line). For contrast, we also plot normal distributions with identical mean and standard deviation (solid black curve), which on logarithmic axes, appear to be similar to a uniform distribution over the range $x \in [1, \langle x \rangle + 3\sigma_x]$. Clearly, career longevity is not in agreement with a traditional Gaussian "bell-curve" pdf. The mean and standard deviation for IPO data is $\langle x \rangle = 1300 \pm 2100$ and for AB data is $\langle x \rangle = 950 \pm 1800$.

We plot in Figures 1 and 2 the probability density function (pdf) of career longevity and success for several statistical categories. We use this common graphical method to illustrate the range and frequency of values that historically occur and to uncover information about the complexity of the underlying system. In Section 1.3 we motivate a detrending method, and provide examples from other fields. In Section 2 we quantify the mathematical averages used to remove the league-wide ability trends that are time-dependent. In Section 3 we discuss both the surprising and the intuitive results of detrending, with some examples of careers that either "rose" or "fell" relative to their traditional rank. In the electronic-only supplementary information (SI) section we present 10 tables listing the Top-50 All-Time ranking of player accomplishments (both career and seasonal) for traditional metrics versus detrended metrics. In Section 3.2 we use the pdf for each statistical category to quantify statistical benchmarks

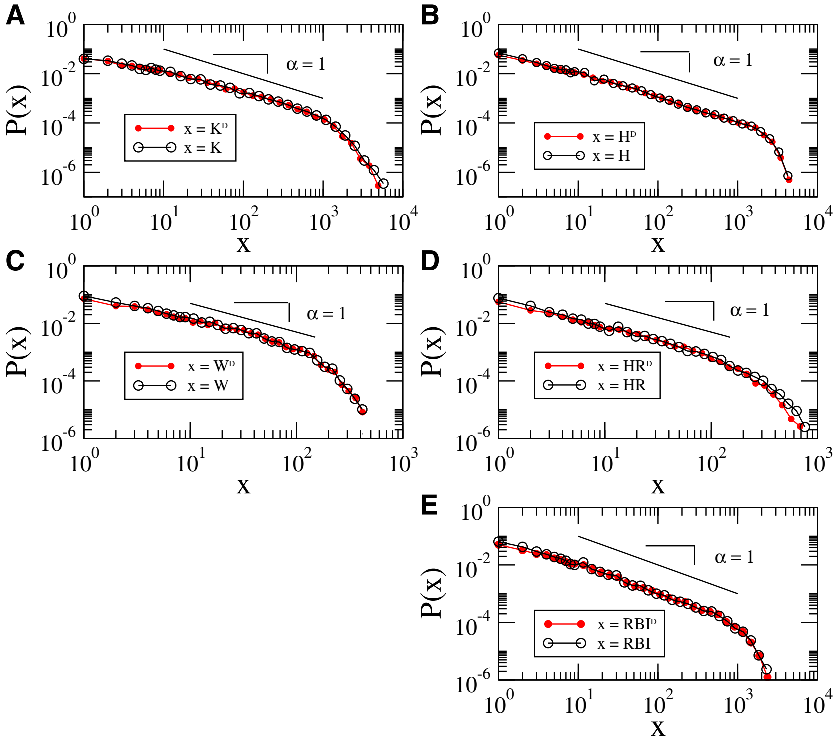

Fig. 2. A comparison of the probability density functions of traditional metrics and detrended metrics. There are small deviations across the entire range between the relative frequency of traditional metrics X (open circles) and detrended metrics X^D (filled circles) for (A) strikeouts K, (B) hits H, (C) wins W, (D) home runs HR, and (E) runs batted in RBI. The data is computed using players ending their career in the 90-year period 1920–2009. The distributions are invariant under detrending. This invariance implies that the statistical regularity observed in the right-skewed distributions for longevity and success in professional sports arises from both the wide range of intrinsic talent among athletes and the underlying nature of competition, and not time-dependent factors which may artificially inflate relative success rates. These extremely right-skewed pdfs for career metrics extend over more than three orders of magnitude. For visual comparison, we show a power-law with exponent $\alpha = 1$ (straight black lines).

that distinguish elite careers, for both traditional and detrended metrics.

1.2 Career longevity

The recent availability of large datasets, coupled with unprecedented computational power, has resulted in many large scale studies of human phenomena that would have been impossible to perform at any previous point in history [8]. A relevant study [9] analyzes the surprising features of career length in Major League Baseball from the perspective of statistical physics and first uncovered the incredibly large disparity between the numbers of "one-hit wonders" – those players who one has probably never heard of and probably never seen – and the "iron horses" – those legends who lasted at the upper tier of professional baseball for several decades and who became household names. Using methods from statistical physics, a recent study [10] quantitatively analyzes the *rich-get-richer* "Matthew Effect"[1] and demonstrates that the universal distribution of career longevity, which is empirically observed in academics and for several professional sports, can be explained by a simple model for career progress.

Career length is the most important statistical quantity for measures of career success in professional sports. This is because all success measures, such as home runs or strikeouts, are obtained in proportion to the number of opportunities that constitute the career length. Furthermore, a player's number of successes is constant in time upon retirement, as opposed to e.g. scientific citations or musical record sales which continue to grow after career termination. In baseball, in the simplest sense, an opportunity is a plate appearance (for batters) or a mound appearance (for pitchers).

In this paper, we only consider the two following metrics for player *opportunity*:

(i) a player's number of at-bats (AB); and
(ii) a pitcher's innings pitched in outs (IPO).

These definitions follow from the unique style of baseball, which is simultaneously a single-opponent game (pitcher versus batter) and a team game (offense versus defense). In each "iteration" of the game of baseball, a batter faces a pitcher, and the outcome of this contest is either an "out" or the advancement of the batter onto the bases. The metrics IPO and AB quantitatively account for these

[1] *"For to all those who have, more will be given, and they will have an abundance; but from those who have nothing, even what they have will be taken away"*. Matthew 25:29, NRSV.

two types of outcomes. Although other definitions for opportunity can be justified, e.g. plate appearances (PA), the salient results are not sensitive to the exact definitions.

We analyze data from Sean Lahman's Baseball Archive [7] which has batting and pitching data for over 17 000 players. In this paper, we do not distinguish between pitchers and fielders in the case of batting statistics, which neglects the appearance variations arising from the designated hitter (DH) rule in the American League, whereby a substitute is allowed to bat for the pitcher during the game[2].

To compare players across all eras, with and without detrending, we utilize the graphical representation of the probability density function (pdf). This is a first step to understanding the frequency of particular types of careers.

The power law pdf

$$P(x) \sim x^{-\alpha}, \quad (1)$$

is found in empirical studies of many complex systems where competition drives the dynamics, with examples ranging from blockbuster Hollywood movies to human sexuality [11–17]. An important feature of the scale-free power law is the large disparity between the most probable value and the mean value of the distribution [18,19], where the most probable value $x_{mp} \sim 1$, while the mean value $\langle x \rangle$ is infinite for $\alpha \leq 2$. This is in stark contrast to the Gaussian (Normal) distribution pdf for which the mean value and the most probable value coincide.

In the case of the career statistics analyzed here, which are extremely right-skewed, analyzing only $\langle x \rangle$ overlooks the incredibly large range of values which includes monumental events that occur relatively frequently compared to the predictions of a Gaussian distribution. Indeed, in the cases of extremely right-skewed distributions, a typical scale for career length is not well-defined.

In order to emphasize the disparity between the long and short careers, consider the ratio of the longest career (Pete Rose, 14 053 at-bats) to the shortest career (many individuals with one at-bat), which is roughly 10 000. For comparison, the ratio of the tallest baseball player (Jon Rauch, 6 feet 11 inches) to the shortest baseball player (Eddie Gaedel, 3 feet 7 inches) is roughly 2. The relatively small value of the player height ratio follows from the properties of the Gaussian distribution, which is well-suited for the description of height in a human population.

The statistical regularity describing power-law behavior with exponent $\alpha \approx 1$ can be roughly phrased as such: for every Mickey Mantle (8102 career at-bats), there are roughly 10 players with careers similar to Doc "the Punk" Gautreaus (806 career at-bats); and for every Doc "the Punk" Gautreau there are roughly 10 players with careers similar to Frank "the Jelly" Jelincich (8 career at-bats with one hit). This statistical property arises from the ratio of frequencies

$$P(x_1)/P(x_2) \sim (x_1/x_2)^{-\alpha}, \quad (2)$$

which only depends on α and the scale-free ratio x_1/x_2. This statistical regularity also applies to the pdfs that quantify career success metrics for hits (H), home runs (HR), runs batted in (RBI), wins (W) and strikeouts (K).

Thus, in power law distributed phenomena, there are rare extreme events that are orders of magnitude greater than the most common events. For the pdfs of career longevity and success analyzed in this paper for which $\alpha < 1$, we observe truncated power-law pdfs

$$P(x) \sim x^{-\alpha} e^{-(x/x_c)}, \quad (3)$$

which have a finite mean $\langle x \rangle$ and standard deviation σ. The truncated power-law distribution captures the surprisingly wide range of career lengths that emerge as a result of the competition for playing time at the top tier of professional sports. The truncation of the scaling regime results primarily from the finite length of a player's career.

1.3 Detrending

Detrending is a common method used to compare observations made at different times, with applications in a wide range of disciplines such as economics, finance, sociology, meteorology, and medicine. Detrending with respect to price inflation in economics is commonly referred to as "deflation" and relies on a consumer price index (CPI). The CPI allows one to properly compare the cost of a candy bar in 1920 dollars to the cost of a candy bar in 2010 dollars. In stock market analysis, one typically detrends intraday volatility by removing the intraday trading pattern corresponding to relatively high market activity at the beginning and end of the market day, which results in a daily activity trend that is "U-shaped". In meteorology, trends are typically cyclical, corresponding to daily, lunar, and annual patterns, and even super-annual patterns as in the case of the El Nino effect. Cyclical trends are also encountered in biological systems, as in the case of protein concentration fluctuations over cell life cycles. In baseball, the trends that we will analyze are those that are associated with player performance ability, or prowess.

It is common for paradigm shifts to change the nature of business and the patterns of success in competitive professions. Baseball has many examples of paradigm shifts, since the game has changed radically since its conception over a century ago. As a result, the relative value of accomplishments depends on the underlying time period. For example, although a home-run will always be a home-run, and a strikeout will always be a strikeout, the rate at which these two events occur has changed drastically over time.

[2] Neglecting the differences between pitchers and fielders does not change the results of this paper, as the pdfs of longevity and success maintain their functional form, even if one distinguishes between pitchers and fielders, as performed in [9]. Furthermore, the only affect of pitchers on batting prowess is to effectively reduce the average by a relatively small percent. Also, in this paper we do not distinguish between complete and incomplete careers, which also contributes only slightly to the distribution of success across all players.

A relevant historical example is the case of Babe Ruth. Before Ruth, home runs were much less frequent than they are in 2010. However, following changes in the rule set accompanied by Babe Ruth's success in the 1920's, many sluggers emerged that are summarily remembered for their home run prowess. The main time-dependent aspect we consider in this paper is the variation in relative player ability, a generic concept that can be easily applied to other professions. Reference [9] finds clear evidence for non-stationarity in the seasonal home-run ability, both on the career and the seasonal level. By comparing the pdfs for career home runs for players belonging to either the 1920–1960 or the 1960–2000 periods, it is shown that the pdf for career home-runs are shifted towards larger totals in the more-recent 1960–2000 period. Moreover, by comparing the pdfs for seasonal home-run ability for players belonging to one of the three periods 1940–1959, 1960–1979, or 1980–2006, it is shown that at the fundamental seasonal time-scale, the home-run rates among players is also changing, where the pdf is becoming more right-skewed with time. These results show why it is important to account for the era-dependence of statistics when comparing career statistical totals.

Yet, this is not the only time-dependent factor that we consider. By detrending, we remove the net trend resulting from many underlying factors, season by season, which allows the proper (statistical) comparison of contemporary players to players of yore (of lore). A significant result of this paper is that detrending for seasonal prowess maintains the overall pdf of success while re-ordering the ranking of player achievements locally. This means that the emergence of the right-skewed pdfs for longevity and success are not due to changes in player ability, but rather, result from the fundamental nature of competition.

The idea behind detrending is relatively straightforward. By calculating the average prowess of all players in a given season, we effectively renormalize all statistical accomplishments to the typical prowess of all contemporaneous competitors. Hence, detrending establishes relative significance levels, such that hitting fifty home runs was of less relative significance during the "Steroids Era" than hitting fifty home runs during the 1920's. The objective of this work is to calculate the detrended statistics of a player's whole career. To this end, we compare career metrics that take into account the time-dependence of league-wide player ability. While there is much speculation and controversy surrounding the causes for changes in player ability, we do not address these individually. In essence, we blindly account for not only the role of PED [20–26], but also changes in the physical construction of bats and balls, sizes of ballparks, talent dilution of players from expansion [27,28], etc.

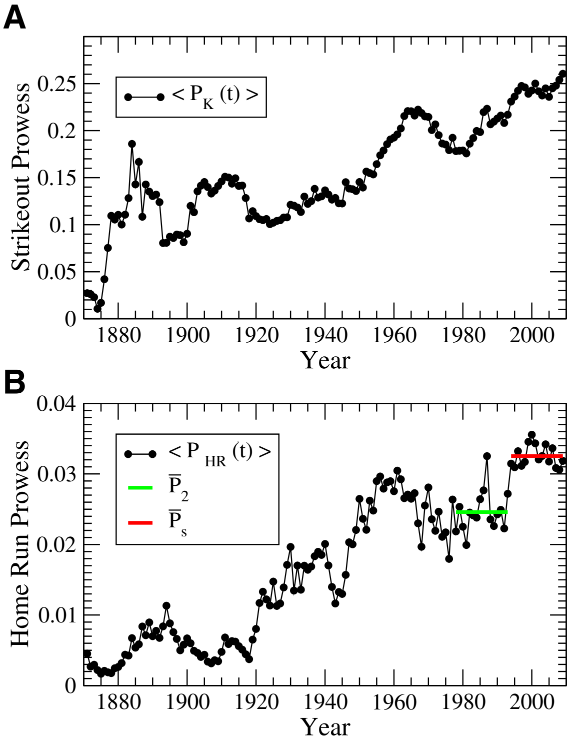

Fig. 3. Success rates reflect the time-dependent factors that can inflate or deflate measures of success. The annual prowess for (A) strikeouts (K) and (B) home runs (HR) are calculated using equation (5). Prowess is a weighted measure of average league wide ability, with more active players having a larger statistical weight than less active players in the calculation of the prowess value $\langle P\rangle$. (B) We also plot the average values $\overline{P}_1$ and $\overline{P}_s$ of $\langle P_{HR}(t)\rangle$ over the 16-year periods $\{Y_1\} \equiv 1978-1993$ and $\{Y_s\} \equiv 1994-2009$, where the latter period roughly corresponds to the "steroids" era. We calculate $\overline{P}_1 = 0.025 \pm 0.003$ and $\overline{P}_s = 0.033 \pm 0.002$, and find $\overline{P}_1 < \overline{P}_s$ at the 0.005 confidence level. For the difference $\Delta \equiv \overline{P}_s - \overline{P}_1$, we calculate the confidence interval $0.005 < \Delta < 0.010$ at the 0.01 confidence level.

2 Materials and methods

2.1 Data

We analyze historical major league baseball (MLB) player data compiled and made publicly available by Sean Lahman [7]. The Lahman Baseball Database is updated at the end of each year, and has player data dating back to 1871. In total, this database records approximately 35 000 players seasons and approximately 17 000 individual player careers.

2.2 Quantifying average prowess

We define prowess as an individual player's ability to achieve a success x (e.g. a home run, strikeout) in any given opportunity y (e.g. an AB or IPO). In Figure 3 we plot the average annual prowess for strikeouts (pitchers) and home runs (batters) over the 133-year period 1876–2009 in order to investigate the evolution of player ability in Major League Baseball. The average prowess serves as an index for comparing accomplishments in distinct years. We conjecture that the changes in the average prowess are related to league-wide factors which can be quantitatively

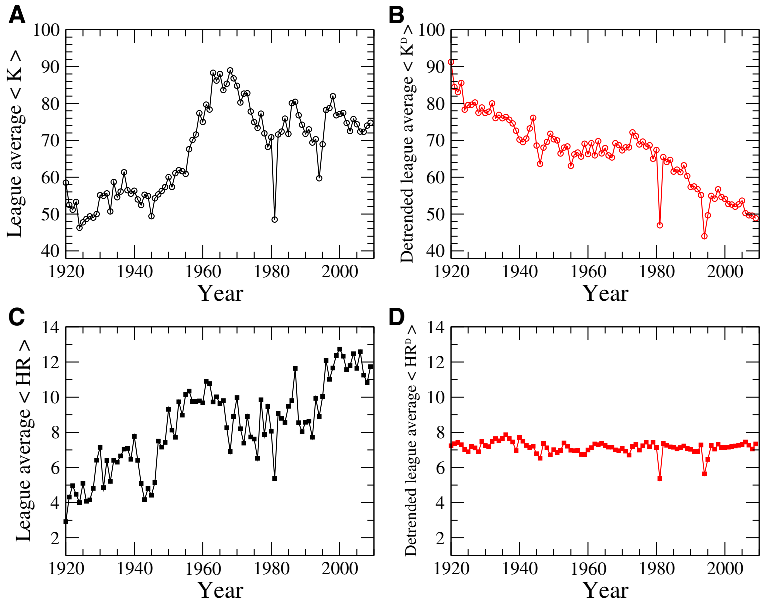

Fig. 4. A comparison of traditional and detrended league averages demonstrates the utility of the detrending method. Annual per-player averages for (A) strikeouts (B) detrended strikeouts (for pitchers), (C) home run, and (D) detrended home runs (for batters). The detrended home run average is remarkably constant over the 90-year "modern era" period 1920–2009, however there remains a negative trend in the detrended strikeout average. This residual trend in the strikeout average may result from the decreasing role of starters (resulting in shorter stints) and the increased role in the bullpen relievers, which affects the average number of opportunities obtained for players in a given season. This follows from the definition of the detrended average given by equation (10). A second detrending for average innings pitched per game might remove this residual trend demonstrated in Figure 5. The sharp negative fluctuations in 1981 and 1994–1995 correspond to player strikes resulting in season stoppage and a reduced average number of opportunities $\langle y(t)\rangle$ for these seasons.

removed (detrended) by normalizing accomplishments by the average prowess for a given season.

We first calculate the prowess $P_i(t)$ of an individual player i as

$$P_i(t) \equiv x_i(t)/y_i(t), \quad (4)$$

where $x_i(t)$ is an individual's total number of successes out of his/her total number of opportunities $y_i(t)$ in a given year t. To compute the league-wide average prowess, we then compute the weighted average for season t over all players

$$\langle P(t)\rangle \equiv \frac{\sum_i x_i(t)}{\sum_i y_i(t)} = \sum_i w_i(t) P_i(t), \quad (5)$$

where

$$w_i(t) = \frac{y_i(t)}{\sum_i y_i(t)}. \quad (6)$$

The index i runs over all players with at least y' opportunities during year t, and $\sum_i y_i$ is the total number of opportunities of all $N(t)$ players during year t. We use a cutoff $y' \equiv 100$ which eliminates statistical fluctuations that arise from players with very short seasons.

We now introduce the detrended metric for the accomplishment of player i in year t,

$$x_i^D(t) \equiv x_i(t)\,\frac{\overline{P}}{\langle P(t)\rangle} \quad (7)$$

where $\overline{P}$ is the average of $\langle P(t)\rangle$ over the entire period,

$$\overline{P} \equiv \frac{1}{110}\sum_{t=1900}^{2009}\langle P(t)\rangle. \quad (8)$$

The choice of normalizing with respect to $\overline{P}$ is arbitrary, and we could just as well normalize with respect to $P(2000)$, placing all values in terms of current "2000 US dollars", as is typically done in economics.

In Figure 4 we compare the seasonal average of $\langle x(t)\rangle$ to the prowess-weighted average $\langle x^D(t)\rangle$, for strikeouts per player and home runs per player. We define $\langle x(t)\rangle$ as

$$\langle x(t)\rangle = \frac{1}{N(t)}\sum_i x_i(t) = \langle P(t)\rangle\frac{\sum_i y_i(t)}{N(t)} = \langle P(t)\rangle\langle y(t)\rangle \quad (9)$$

and $\langle x^D(t)\rangle$ as,

$$\langle x^D(t)\rangle = \frac{1}{N(t)}\sum_i x_i^D(t) = \frac{\overline{P}}{\langle P(t)\rangle N(t)}\sum_i x_i(t) = \overline{P}\,\frac{\langle x(t)\rangle}{\langle P(t)\rangle} = \overline{P}\,\langle y(t)\rangle. \quad (10)$$

As a result of our detrending method defined by equation (7), which removes the time-dependent factors that

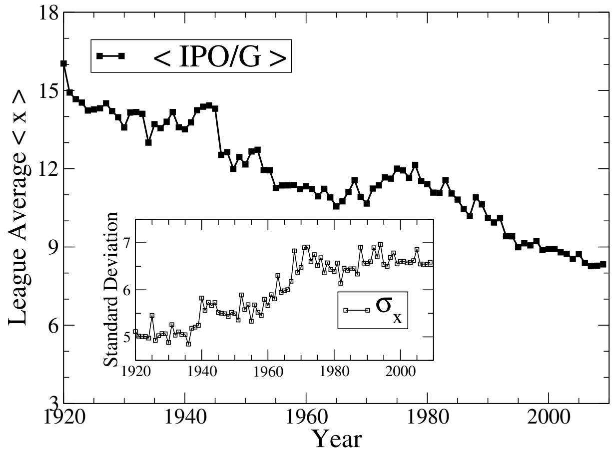

Fig. 5. Annual per-player average for innings pitched per game appearances (IPO/G) demonstrates strategy change in pitcher use. A clear decreasing trend in the average number of innings pitched per outing demonstrates the increased role of relief pitchers. This is a strategic trend, which we do not address in this paper, as we have restricted our analysis to player ability trends. This explains why detrending for pitching ability (see Fig. 4) does not yield a constant average over time (as in the case with detrended home-run ability). We use a cutoff $y' =$ 10 games in computing this average.

affect league-wide ability from the average number of successes across all players in a given season, we find that equation (10) is independent of $\langle P(t)\rangle$. The averages computed in equations (9) and (10) are computed for players with $y_i(t) > y' \equiv 100$ appearances and $x_i(t) > x' \equiv 0$ successes, to eliminate statistical fluctuations arising from insignificant players.

Using this method, we calculate detrended metrics for baseball players for both single season (x_i^D corresponding to Eq. (7)) and total career accomplishments (X_i^D corresponding to Eq. (11)). Naturally, the detrended career metric is cumulative, which we calculate for each player i over his career as

$$X_i^D \equiv \sum_{s=1}^{L} x_i^D(s), \qquad (11)$$

where s is the season index and L is the player's career length measured in seasons.

3 Results

3.1 Comparing across historical eras

In this section we discuss the results of detrending. We begin with the analysis of career longevity, which we discuss briefly, and refer interested readers to [9] for a more detailed discussion. The main take-home result from Figure 1 is the variety of career length of major league players. Surprisingly, we find that about 3% of fielding batters (non-pitchers) have their premier and finale in the same AB. Furthermore, approximately 5% of all fielding batters finish their career with only one hit. Similarly, 3% of all pitchers complete their career with an inning or less of pitching. Yet, remarkably, there are several players with careers that span more than 2000 games, 10 000 at bats, and 4000 innings. This incredible range is captured by the pdfs of career longevity which appear as linear when plotted on log-log scale.

The statistical regularity of the pdfs in Figures 1 and 2 allows for the quantitative comparison of careers. Furthermore, this statistical regularity is *very* different from the statistical regularity captured by the common Gaussian (Normal) "bell curve". For comparison, we plot Gaussian pdfs in Figure 1 which have the same mean and standard deviation as the data represented by the power law pdf. On a log-log scale, the corresponding Gaussian pdfs appear as uniformly distributed up until a sharp cutoff around 10^4 appearances. The striking disagreement between the data and the Gaussian bell-curves is evidence of the complexity underlying career longevity. The truncated power-law, defined in equation (3), has two parameters, α and x_c, where we observe values $\alpha \lesssim 1$ for all metrics studied. The value of x_c marks the beginning of the exponential cutoff which separates the exceptional careers for which $x > x_c$ from the more common careers corresponding to $x < x_c$.

In Figures 2A–2E we plot the pdfs for several statistical categories, and for each we plot both the traditional and detrended metrics. We observe only a slight variation between the traditional and detrended pdfs, on a log-log scale. This is an indication that the detrending method only makes relatively small refinements to the career totals. For a given player, the ratio between his detrended and traditional metrics $r \equiv X^D/X$ is closely distributed around a value of unity for all metrics analyzed (see Tab. 1). Interestingly, for the case of career home runs, the distribution of r-values across all players is bimodal, with $\langle r\rangle = 1.2 \pm 1.0$, which accounts for the the slight deviation in the tail of the pdf for detrended home runs in Figure 2D.

In Figure 3 we plot the trends in average prowess $\langle P(t)\rangle$, defined in equation (5), for pitchers obtaining strikeouts and batters hitting home runs over the years 1876–2009. The number of players $N(y')$ entering into the calculation of each data point depends on the cutoff y' and varies according to the completeness of the database and to the number of players on MLB rosters (see Fig. 6). In the case of both home runs and strikeout ability, there is a marked increase with time reflecting the general changes in game play, player ability, and the competitive advantage between pitcher and batter. For pitchers, there are two significant periods corresponding to the dead-ball era (approximately 1900–1920) and to the era around the 1960's when pitchers were relatively dominant (ending when the mound was officially lowered in 1969 by league wide mandate). The period beginning in the 1920's shows a clear increase in home run ability, often attributed to the outlawing of the spitball and the emergence of popular sluggers, such as Babe Ruth and Rogers Hornsby. Together, Figures 3A and 3B indicate that the average prowess of major league players is non-stationary.

Table 1. Parameter values for the pdfs of longevity and career metrics X and detrended career metrics X^D defined in equation (11). The values of α and $x_c = \langle x\rangle/(1-\alpha)$ are calculated using the gamma distribution maximum likelihood estimator. The values of the average and standard deviation $\langle x\rangle \pm \sigma$ corresponds to actual data, while $\sigma_\Gamma = x_c\sqrt{1-\alpha}$ is the standard deviation of the gamma distribution Gamma$(x;\alpha,x_c)$. The benchmark value x^* can be approximated as $x^* \approx 4\ \sigma_\Gamma$ for the $f = 0.02$ significance level [10]. We also compute the value of x^* for the $f = 0.01$ significance level which we list in parenthesis. The first column corresponds to the average and standard deviation $\langle r\rangle \pm \sigma$ of the ratio $r \equiv X^D/X$ between an individual's detrended and traditional career success values.

Statistic x	$\langle r\rangle \pm \sigma$	$\langle x\rangle \pm \sigma$	α_{MLE}	x_c	σ_Γ	x^*
AB		950 ± 1800	0.68	2900	1700	6500 (8000)
IPO		1300 ± 2100	0.57	3000	2000	7800 (9500)
K		260 ± 430	0.55	570	390	1500 (1800)
K^D	1.0 ± 0.3	240 ± 400	0.55	520	350	1400 (1700)
H		290 ± 530	0.65	820	490	1900 (2300)
H^D	1.0 ± 0.04	290 ± 530	0.65	820	490	1900 (2300)
W		34 ± 49	0.40	55	43	170 (200)
W^D	1.0 ± 0.01	34 ± 49	0.40	55	43	170 (200)
HR		42 ± 78	0.53	89	62	240 (290)
HR^D	1.2 ± 1.1	36 ± 65	0.52	74	51	200 (240)
RBI		150 ± 280	0.61	390	240	940 (1200)
RBI^D	1.0 ± 0.1	150 ± 270	0.61	380	240	900 (1100)

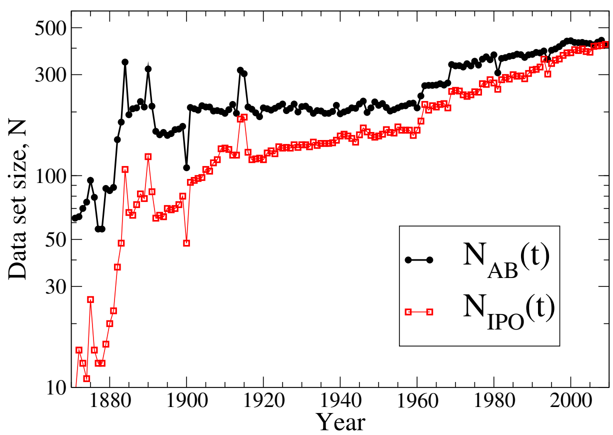

Fig. 6. Semi-log plot of data set size exhibits the growth of professional baseball. The size of the data sets, N, is used to compute the annual trends for pitchers and batters in Figures 3 and 4. These data sets correspond to cutoffs $x' \equiv 0$ and $y' \equiv 100$ in equations (5)–(10). Interestingly, we observe a spike in league size during WWI, possibly corresponding to the widespread replacement of league veterans with multiple replacements through the course of the season. We also note that one can clearly see the jumps associated with the expansion between 1960 and 1980.

Closely related, but not equal to average prowess $\langle P(t)\rangle$, is the average number of successes $\langle x\rangle$ (e.g. home runs, hits, strikeouts, wins, etc.) per player for a particular season. Comparing equations (9) and (10), the resulting relationship $\langle x^D(t)\rangle \equiv \langle y(t)\rangle\overline{P}$ may appear trivial, since the quantity $\langle x^D(t)\rangle$ does not depend on yearly player abilities, but rather yearly player opportunities $\langle y(t)\rangle$. This is entirely the motivation for detrending: we aim to remove all factors contributing to league-wide player ability. Thus by effectively removing the seasonal trends from each player's metrics we measure accomplishments based only on comparable opportunities. We graphically illustrate this result in Figure 4 where we plot the average number $\langle x\rangle$ of strikeouts and home runs next to the detrended average number $\langle x^D\rangle$ of strikeouts and home runs, per pitcher and batter, respectively, over the years 1920–2009.

We also observe in Figure 4C an approximately linear decrease in the detrended league average for strikeouts over time. This is a result of the increased use of relief pitchers (pitchers who do not regularly start games, but rather replace the starting pitchers on demand) in baseball, which reduces the number of innings pitched per pitcher. This, however, is a strategic change in the way the game of baseball is managed, and does not reflect player ability. In order to detrend for this secondary effect, which might make possible a comparison between Cy Young, Greg Maddux and Mariano Rivera, one must further detrend pitching statistics by the league average of innings per game, which we illustrate in Figure 5. The increase in the standard deviation of IPO/G over time is due to the increased variability in the type and use of pitchers in baseball.

The relatively constant value for the detrended league average of home-runs in Figure 4D demonstrates that this method properly normalizes home-run statistics across time. As a result, the baseline for seasonal comparison is approximately 7 home runs per season over the entire 90-year period 1920–2009. We use this relatively constant baseline to correctly compare the single-season accomplishments of Babe Ruth, in each of his historical seasons 1920–1933, with the single-season accomplishments of Roger Maris' 61 in '61, with the wild race between Mark McGwire and Sammy Sosa in 1998, and with Barry Bonds' pinnacle performance in 2001. In Table S6 we rank the top-50 individual home run performances by season, both for the traditional and detrended metrics. Overwhelmingly, Babe Ruth's accomplishments in the 1920's are superior to those of his peers. Surprisingly, there are no modern players later than 1950 that make the top-50 list, not even

Table 2. Ranking of Career Home Runs (1871–2009). The left columns lists the traditional ranking of career statistics, where the top 25 players are ranked along with their final season (career length in seasons listed L in parenthesis) and their career metric tally. The right columns list the detrended ranking of career statistics $Rank^*$, where the corresponding traditional ranking of the player is denoted in parenthesis.

Traditional rank				Detrended rank				
Rank	Name	Final season (L)	Career metric	Rank*(Rank)	% Change	Name	Final season (L)	Career metric
1	Barry Bonds	2007 (22)	762	1(3)	66	Babe Ruth	1935 (22)	1215
2	Hank Aaron	1976 (23)	755	2(23)	91	Mel Ott	1947 (22)	637
3	Babe Ruth	1935 (22)	714	3(26)	88	Lou Gehrig	1939 (17)	635
4	Willie Mays	1973 (22)	660	3(17)	82	Jimmie Foxx	1945 (20)	635
5	Ken Griffey Jr.	2009 (21)	630	5(2)	–150	Hank Aaron	1976 (23)	582
6	Sammy Sosa	2007 (18)	609	6(124)	95	Rogers Hornsby	1937 (23)	528
7	Frank Robinson	1976 (21)	586	7(192)	96	Cy Williams	1930 (19)	527
8	Alex Rodriguez	2009 (16)	583	8(1)	–700	Barry Bonds	2007 (22)	502
8	Mark McGwire	2001 (16)	583	9(4)	–125	Willie Mays	1973 (22)	490
10	Harmon Killebrew	1975 (22)	573	10(18)	44	Ted Williams	1960 (19)	482
11	Rafael Palmeiro	2005 (20)	569	11(13)	15	Reggie Jackson	1987 (21)	478
12	Jim Thome	2009 (19)	564	12(14)	14	Mike Schmidt	1989 (18)	463
13	Reggie Jackson	1987 (21)	563	13(7)	–85	Frank Robinson	1976 (21)	444
14	Mike Schmidt	1989 (18)	548	14(10)	–40	Harmon Killebrew	1975 (22)	437
15	Manny Ramirez	2009 (17)	546	15(577)	97	Gavvy Cravath	1920 (11)	433
16	Mickey Mantle	1968 (18)	536	16(718)	97	Honus Wagner	1917 (21)	420
17	Jimmie Foxx	1945 (20)	534	17(18)	5	Willie McCovey	1980 (22)	417
18	Ted Williams	1960 (19)	521	18(557)	96	Harry Stovey	1893 (14)	413
18	Frank Thomas	2008 (19)	521	19(5)	–280	Ken Griffey Jr.	2009 (21)	411
18	Willie McCovey	1980 (22)	521	20(28)	28	Stan Musial	1963 (22)	410
21	Eddie Mathews	1968 (17)	512	21(28)	25	Willie Stargell	1982 (21)	399
21	Ernie Banks	1971 (19)	512	22(25)	12	Eddie Murray	1997 (21)	397
23	Mel Ott	1947 (22)	511	22(8)	–175	Mark McGwire	2001 (16)	397
24	Gary Sheffield	2009 (22)	509	24(16)	–50	Mickey Mantle	1968 (18)	394
25	Eddie Murray	1997 (21)	504	24(113)	78	Al Simmons	1944 (20)	394

the career years of George Foster ($x^D = 40$ in 1977), Barry Bonds ($x^D = 42$ in 2001), and Mark McGwire ($x^D = 44$ in '98). Even the 61 home runs of Roger Maris in 1961 is discounted to $x^D = 40$ detrended home runs, placing him in a tie for 97th on the detrended ranking list.

In fact, in 1961, the season was 8 games longer than in 1927, when Babe Ruth hit a seemingly insurmountable 60 home runs. There was much public resentment over Babe Ruth's seminal record being broken, which caused the commissioner of baseball in 1961 to suggest placing an asterisk next to Roger Maris' record [5]; In analogy, it is being suggested that the asterisk be used to denote the conditional achievement of baseball players found guilty of using PED. Interestingly, after the 1961 season, the commissioner reacted to the sudden increase in home runs by expanding the strike zone by league mandate, which resulted in the competitive balance tipping in favor of the pitchers during the 1960's. On several historical occasions, the competitive balance was shifted by such rule changes. Nevertheless, the detrending method accounts for variations in both season length and various performance factors, since this method normalizes achievement according to local seasonal averages. The results of detrending may be surprising to some, and potentially disenchanting to others. We note that these results *do not* mean that Babe Ruth was a better at hitting home runs than any other before or after him, but rather, *relative* to the players during his era, he was the best home run hitter, by far.

Table 2 lists the top-25 career home-run rankings, and Tables S1–S10 (in the electronic-only supplementary information) list the top-50 rankings for several other statistical categories. In each table, we compare the traditional rankings alongside the detrended rankings. Tables S1–S3 (in the electronic-only supplementary information) list the career batting statistics for HR, H and RBI, while Tables S4 and S5 (in the electronic-only supplementary information) list career pitching statistics for strikeouts K and wins W. We choose these statistics because four of these categories have a popular benchmark associated with elite careers, the 500 home run, 3000 hit, 3000 strikeout and 300 win "clubs".

Each table provides two parallel rankings, the traditional rank and the detrended rank. The column presenting the traditional rank lists from left to right the traditional ranking, the player's name, the player's final season, with the total number of seasons played (L), and the career total X. The column presenting the detrended rank lists from left to right the detrended ranking $Rank^*$ with the corresponding traditional rank ($Rank$) in parenthesis, the relative percent change in the rankings $\%Change = (Rank^* - Rank)/Rank$, the player's name, the player's final season with the total number of seasons (L), and the detrended career total X^D corresponding to equation (11). The tables listing seasonal records have an analogous format, except that the year listed is the year of the achievement with the number of seasons into his career ($Y\#$).

In the case of career home runs (see Tabs. 2 and S1), Babe Ruth towers over all other players with almost twice as many detrended career home runs as his contemporary, the third-ranked Lou Gehrig. Hank Aaron's detrended career total is discounted by a significant factor and he falls to 5th place. Overall, the eras become well-mixed for detrended home runs, whereas the traditional top-50 is dominated by players from the past thirty years. Possibly the most significant riser in the detrended homer run rankings is Cy Williams, who played in the era prior to Babe Ruth. Cy Williams moves up largely due to the relatively low home run prowess of the early 1900's. Nevertheless, according to our statistical analysis, Cy Williams, along with Gavvy Cravath, are strong candidates for retroactive induction into the Hall of Fame. A similar amount of mixing occurs for the career strikeouts rankings. Of particular note are Amos Rusie and Dazzy Vance, titans from a distant era whose accomplishments rise in value relative to the contemporary household names of Nolan Ryan, Randy Johnson, and Roger Clemens.

In contrast to the vast reordering of the top 50 for home runs and strikeouts, the detrended rankings for career hits undergoes much less change. This indicates that hitting prowess has been relatively stable over the entire history of baseball. In light of this fact, the batting average (BA), which is approximately a hit per opportunity average, is likely an unbiased and representative metric for a player, even without detrending time-dependent factors.

The detrended rankings for career wins show the least amount of change of the 5 statistical categories we studied. We attribute this stability mostly to the nature of a pitcher earning a win, which is significantly related to the overall team strength [28,29]. Since a win results from a combination of factors that are not entirely attributed to the prowess of the pitcher, detrending does not change the relative value of wins in a player-to-player comparison. Wins are also biased towards starting pitchers who, on average, pitch more innings than relievers. The traditional cumulative metrics for pitching undervalue the accomplishments and role of relief pitchers, that have been an increasingly important feature of the game as illustrated in Figure 5. In order to incorporate middle and late relief pitchers into an all-inclusive comparison of pitchers, a metric such as strikeouts per inning or earned run average (ERA) will be more descriptive than career strikeouts or wins. To this end, strikeouts per inning and ERA are fundamentally averages, and would fall into a separate class of pdf than the truncated power-laws observed for the metrics considered in this paper.

In Tables S6–S10 (in the electronic-only supplementary information) we list the traditional and detrended single season statistics for HR, H, RBI, and K. There are too many players to discuss individually, so we mention a few interesting observations. First, the rankings for detrended home runs HR^D are dominated by seasons prior to 1950. Second, of contemporary note, Ichiro Suzuki's single season hits record in 2004, which broke the 83-year record held by George Sisler, holds its place as the top single-season hitting performance of all time. Finally, we provide two top-50 tables for single season strikeouts in Tables S9 and S10 (in the electronic-only supplementary information). We provide two separate tables because the relative performance of pitchers from the 1800's far surpasses the relative performance of contemporary legends. Hence, in Table S9 we rank all single-season performances from 1883–2009, while in Table S10 (in the electronic-only supplementary information) we rank all single-season performances from the "modern" era 1920-present. We note that the dominance of Table S9 (in the electronic-only supplementary information) by 19th century baseball players could reflect fluctuations from small data set size and incomplete records of pitchers in the 19th century (see Fig. 6). Still, Matt Kilroy's 513 strikeouts in 1889 seems unfathomable by today's standards, and the seemingly out of place number may reflect factors which detrending can not account for, e.g. the level of competition being significantly reduced, as baseball was not a full-time profession for many players in the 19th century. Table S10 (in the electronic-only supplementary information), filled with names that are much more familiar, better illustrates the relative merits of forgotten names such as Dazzy Vance and Bob Feller. While the relative changes are mostly positive, with Nolan Ryan's six monumental seasons still notable, there is an unexpected discount of Sandy Koufax's 382 strikeouts in 1965.

3.2 Calculating career benchmarks

In this section we outline an approach for calculating a set of statistical criterion that can be used to objectively define extraordinary careers. Again, we use historical examples from baseball to illustrate the utility of quantifying benchmarks that can be used for distinguishing outstanding performance for professional reward, such as annual bonus, salary increase, tenure.

Due to the rarity of careers surpassing the benchmarks of 500 home runs, 3000 hits, 3000 strikeouts, and 300 wins, these milestones are usually accepted by baseball fans and historians as clear indicators of an extraordinary career. However, these benchmarks are fundamentally arbitrary, and their continued acceptance can probably be attributed to their popularity with the media personalities that cover baseball. Using the properties of the pdf, that we have shown accurately characterizes many baseball statistics, we can extract more objective benchmarks as we now discuss.

We approximate the pdf $P(x)$ of each success metric, x, by the gamma distribution,

$$P(x)dx \approx \text{Gamma}(x; \alpha, x_c)dx = \frac{(x/x_c)^{-\alpha} e^{-x/x_c}}{\Gamma(1-\alpha)} \frac{dx}{x_c} \propto x^{-\alpha}\ e^{-x/x_c}, \qquad (12)$$

and use the mathematical properties of this function in order to define a statistically significant benchmark x^*. We calculate the value of x^* by using the integral properties of the Gamma distribution. For a threshold level f, we determine the value of x^* such that only f percent of players exceed the benchmark value x^*,

$$f = \int_{x^*}^{\infty} \frac{x^{-\alpha} e^{-x/x_c}}{x_c^{1-\alpha} \Gamma(1-\alpha)} dx$$
$$= \frac{\Gamma[1-\alpha, \frac{x^*}{x_c}]}{\Gamma(1-\alpha)} = Q\left[1-\alpha, \frac{x^*}{x_c}\right], \qquad (13)$$

where $\Gamma[1-\alpha, \frac{x^*}{x_c}]$ is the incomplete gamma function and $Q[1-\alpha, \frac{x^*}{x_c}]$ is the regularized gamma function. The regularized gamma function is numerically invertible. Exploiting this property, we calculate

$$x^* = x_c Q^{-1}[1-\alpha, f], \qquad (14)$$

using the inverse regularized gamma function found in standard computing packages. In addition to the analysis performed in [10], where a graphical method is used to determine the values α and x_c from the pdf using a graphical least-squares routine, here we use the maximum likelihood estimator (MLE) in order to determine α and x_c from the observed values, x_i, of the entire data set [30]. The values of α and x_c that maximize the log-likelihood of observing the data set $\{x\}$ of size N are, to first approximation,

$$\alpha_{MLE} = 1 - \frac{3 - z + \sqrt{(z-3)^2 + 24z}}{12z}, \qquad (15)$$

where z is calculated from the N individual data values x_i,

$$z = \ln\left(\sum_{i=1}^{N} x_i\right) - \left(\sum_{i=1}^{N} \ln x_i\right) / N. \qquad (16)$$

Since the average value of the gamma distribution is $\langle x \rangle = (1-\alpha)x_c$, the MLE for x_c is computed using the estimated value of α,

$$x_c = \frac{\langle x \rangle}{1-\alpha} = \frac{1}{N(1-\alpha_{MLE})} \sum_{i=1}^{N} x_i. \qquad (17)$$

Table 1 lists the values of α_{MLE}, x_c, and the corresponding standard deviation $\sigma_\Gamma = x_c\sqrt{1-\alpha}$ calculated for the Gamma pdf. We use a threshold of 2%, i.e. $f = 0.02$, which corresponds roughly to the percentage of all baseball players elected into the Cooperstown Baseball Hall of Fame [10], and then we find the benchmark value can be approximated as $x^* \approx 4\ \sigma_\Gamma = 4\ x_c\ \sqrt{1-\alpha}$. This approximation is a consequence of the universal scaling form of the gamma function $\mathrm{Gamma}(x; \alpha, x_c) = \mathrm{Gamma}(x/x_c; \alpha)$, such that for a given f and α, the ratio

$$\frac{x^*}{\sigma_\Gamma} = \frac{Q^{-1}[1-\alpha, f]}{\sqrt{1-\alpha}} \qquad (18)$$

is independent of x_c. Furthermore, this approximation is valid for all MLB statistics since α is approximately the same for all pdfs analyzed. In Table 1 we compute x^* for both traditional X metrics and detrended X^D metrics at the two thresholds $f = 0.02$ and $f = 0.01$. We see that the values of α_{MLE} are approximately equal for both traditional and detrended data sets, and that the values of x^* do not vary significantly. As a result (at the $f = 0.02$ level), a player with either $X = 1900$ hits or $X^D = 1900$ detrended hits is statistically stellar in comparison to other players.

4 Discussion

The statistical physics of social phenomena is a growing field which aims to describe the complex behavior that emerges from the interactions of agents [10,13,31–35]. While it is often difficult to account for the interaction complexity in explicit governing equations, a first step towards understanding the underlying social mechanism is to study the macroscopic behavior of the system. The quantitative analysis of human achievement in competitive arenas, e.g. sports and academia, is an open topic of investigation, which in recent studies has combined methods from sociology, economics, and statistical physics [9,36–38].

Here we analyze the distribution of success in a population of competitive athletes. Because professional baseball has such a standard and precise method for recording historical achievement, the box-score, we are able to compare the achievements of professional baseball players for over 100 years of MLB. In order to account for changes in relative player ability over time, we have developed a detrending method which accounts for inflationary and deflationary factors and allows for an objective comparison of players across time. Remarkably, we find using our detrending method, that the distributions of career success are invariant after we normalize accomplishments to local averages. Thus, even controlling for time dependent factors, the distribution of career achievement is extremely right-skewed, and can be quantified by a truncated power-law. Furthermore, in order to distinguish stellar careers, we derive non-arbitrary statistical benchmarks based on significance thresholds defined by the pdfs of success for the entire population.

Typically, only the greatest career achievements are recorded in the annals of history. Here we analyze all participants in a competitive system to compare and contrast the various types of careers. The statistical regularity of the pdfs quantifying metrics for career longevity and career success also exist for athletes in several other professional sports leagues [10] (American basketball, Korean baseball, English football), as well as for research scientists [36]. A surprising observation is the large numbers of "one-hit wonders", along with much smaller, but statistically significant and theoretically predictable, number of stellar "iron-horse" careers within these competitive professions. We find a surprising statistical regularity which bridges the gap between the large number of individuals with very few career accomplishments and the few individuals with legendary career accomplishments. This empirical law emerges as a result of analyzing the entire population of agents/participants. Furthermore, by analyzing the success rates across an entire population of agents, we quantify the time dependence of trends which alter the relative significance of individual achievements. Analogous efforts are taking place in the bibliometric sciences in order to establish universal citation metrics which account for variations across time and academic discipline [36,39].

We demonstrate the utility of our detrending method by accounting for the changes in player performance over time in professional baseball, which is particularly relevant to the induction process for HOF and to the debates regarding the widespread use of PED in professional sports. There has also been debate over the use of cognitive enhancing drugs in academia [40,41]. In baseball, we find a significant increase in home run rates in recent years. Analyzing home run prowess, we find a statistically significant 32% increase in home run rates over the most recent 16-year "steroid era" period 1994–2009 when compared to the previous 16-year period 1978–1993 (see Fig. 3). Hence, the raw accomplishments of sluggers during the steroids era will naturally supersede the records of sluggers from prior eras. So how do we ensure that the legends of yesterday do not suffer from historical deflation? With the increased use of sophisticated sabermetric statistics in baseball and the recent application of network science methods to quantify the extremely large number of head-to-head match-ups between pitcher and batter [42], a new picture of baseball is emerging [43] which views all-time achievement in new light, and is providing an objective framework for comparing achievements across time. In this paper, we consider the most natural measures for accomplishment, the statistics that are listed in every box-score and on every baseball card, in hope that the results are tangible to any historian or fan who is interested in reviewing and discussing the "all-time greats".

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References

1. J. Duch, J.S. Waitzman, L.A.N. Amaral, PLoS ONE **5**, e10937 (2010)
2. G. Berthelot, V. Thibault, M. Tafflet, S. Escolano, N. El Helou et al., PLoS ONE **3**, e1552 (2008)
3. F.D. Desgorces, G. Berthelot, N. El Helou, V. Thibault, M. Guillaume et al., PLoS ONE **3**, e3653 (2008)
4. M. Guillaume, N.E. Helou, H. Nassif, G. Berthelot, S. Len et al., PLoS ONE **4**, e7573 (2009)
5. G.C. Ward, K. Burns, *Baseball: An Illustrated History* (Knopf, New York, 1994)
6. G.J. Mitchell, *Report to the Commissioner of Baseball of an Independent Investigation into the Illegal Use of Steroids and other Performance Enhancing Substances by Players in Major League Baseball* (Office of the Commissioner of Baseball, 2007)
7. Sean Lahman's Baseball Archive: http://baseball1.com/index.php
8. D. Lazer et al., Science **323**, 721 (2009)
9. A.M. Petersen, W-S. Jung, H.E. Stanley, Europhys. Lett. **83**, 50010 (2008)
10. A.M. Petersen, W-S. Jung, J.-S. Yang, H.E. Stanley, Quantitative and empirical demonstration of the Matthew effect in a study of career longevity, accepted for publication, Proc. Natl. Acad. Sci. USA ArXiv preprint: 0806.1224 [physics]
11. V. Pareto, *Cours d'Économie Politique* (Droz, Geneva, 1896)
12. S. Redner, Eur. Phys. J. B **4**, 131 (1998)
13. R.N. Mantegna, H.E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, Cambridge, 1999)
14. R. Albert, H. Jeong, A.-L. Barabási, Nature **401**, 130 (1999)
15. F. Liljeros et al., Nature **411**, 907 (2001)
16. J.A. Davies, Eur. Phys. J. B **27**, 445 (2002)
17. S. Sinha, S. Raghavendra, Eur. Phys. J. B **42**, 293 (2004)
18. M.E.J. Newman, Contemp. Phys. **46**, 323 (2005)
19. A. Clauset, C.R. Shalizi, M.E.J. Newman, SIAM Rev. **51**, 661 (2009)
20. J.J. Koch, Pediatr. Rev. **23**, 310 (2002)
21. T.D. Noakes, N. Engl. J. Med. **351**, 847 (2002)
22. I. Waddington et al., Br. J. Sports Med. **39**, e18 (2005)
23. J.C. Bradbury, *What Really Ruined Baseball* (New York Times, New York, 2007)
24. C.R. McHenry, Surgery **142**, 785 (2007)
25. R.G. Tobin, Am. J. Phys. **76**, 15 (2008)
26. B.J. Schmotzer, J. Switchenko, P.D. Kilgo, Journal of Quantitative Analysis in Sports **4**, 4 (2008)
27. J. Starks, *Homers, homers, homers* (ESPN.com, May 15, 2000)
28. E. Ben-Naim, F. Vazquez, S. Redner, Journal of Quantitative Analysis in Sports **2**, 1 (2006)
29. C. Sire, S. Redner, Eur. Phys. J. B **67**, 473 (2009)
30. S.C. Choi, R. Wette, Technometrics **11**, 683 (1969)
31. P. Holme, C.R. Edling, F. Liljeros, Social Networks **26**, 155 (2004)
32. J.D. Farmer, M. Shubik, E. Smith, Physics Today **58**, 37 (2005)
33. B.F. de Blasio, A. Svensson, F. Liljeros, Proc. Natl. Acad. Sci. USA **104**, 10762 (2007)
34. M.C. González, C.A. Hidalgo, A.-L. Barabási, Nature **453**, 779 (2008)
35. C. Castellano, S. Fortunato, V. Loreto, Rev. Mod. Phys. **81**, 591 (2009)
36. A.M. Petersen, F. Wang, H.E. Stanley, Phys. Rev. E **81**, 036114 (2010)
37. J.E. Hirsch, Proc. Natl. Acad. Sci. USA **102**, 16569 (2005)
38. A.M. Petersen, H.E. Stanley, S. Succi, Statistical regularities in the rank-citation profile, under review
39. F. Radicchi, S. Fortunato, C. Castellano, Proc. Natl. Acad. Sci. **105**, 17268 (2008)
40. B. Maher, Nature **452**, 674 (2008)
41. B. Sahakian, S. Morein-Zamir, Nature **450**, 1157 (2007)
42. S. Saavedra, S. Powers, T. McCotter, M.A. Porter, P.J. Mucha, Physica A **390**, 1131 (2010)
43. J. Click et al., *Baseball Between the Numbers.: why everything you know about the game is wrong* (Basic Books, New York, 2006)