

# Special analytical properties of ultrastrong coherent fields

H.R. Reiss<sup>1,2,a</sup>

<sup>1</sup> Max Born Institute, 12489 Berlin, Germany

<sup>2</sup> American University, Washington, DC 20016-8058, USA

Received 11 November 2008

Published online 13 February 2009 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2009

**Abstract.** Emerging ultrastrong-laser capabilities that can reveal details of vacuum structure have intensified research into the fundamentals of quantum electrodynamics. It has been more than half a century since relativistic nonperturbative methods were introduced into the study of strong-field phenomena. Much of the early progress remains of fundamental relevance, but is known to only a small group of researchers. The aim of this paper is to reveal some of that work and to show how it impacts on current investigations. A basic result is that it has been shown that strong, single-mode fields (i.e. laser fields) can be treated by relativistic quantum mechanics with results identical to fully quantized electrodynamics. Attention is drawn to the existence of a Volkov Green's function that has a clear physical interpretation as predicting several series of relativistic Floquet sideband states. It is more transparent and informative than the Volkov Green's function of Schwinger. It is also shown that the fundamental experiments performed at the Stanford Linear Accelerator Center in 1997 on photon-multiphoton pair production could not be a high-order perturbative result, as was presumed by the investigators. The intensity employed was beyond the radius of convergence of perturbation theory, and the seeming perturbative increase in rate with intensity is an artifact. Of particular significance is the demonstration that a free electron in a strong plane-wave field (a "Volkov electron") exists in an intensity-dependent superposition of angular momentum states, and is no longer a simple spin-1/2 particle.

**PACS.** 12.20.-m Quantum electrodynamics – 14.60.Cd Electrons – 13.40.-f Electromagnetic processes and properties

## 1 Introduction

The expectation of major advances in laser capabilities has placed a new focus on the fundamentals of quantum electrodynamics (QED). An often-mentioned goal is to approach the "Schwinger limit" [1], where the electric field of the laser would, in itself, be sufficient to create electron-positron pairs from the vacuum. (With the proviso that a pure plane wave cannot, by itself, create pairs.) More generally, there is renewed interest in all aspects of the structure of the vacuum. QED occupies a special place among the methods available to physics. It has proven itself to be capable of remarkable quantitative agreement between theoretical predictions and precise laboratory measurements, as exemplified by the anomalous magnetic moment of the electron, where agreement now extends to one part in  $10^{12}$  [2,3]. Nevertheless, fundamental questions remain.

After its initial establishment in the late 1920s and early 1930s, QED acquired important new theoretical power about twenty years later when covariant methods were introduced. Much of that work centered around the late John A. Wheeler and his students, including both the achievements of the famous Richard P. Feynman and the

lesser-known activities of John S. Toll. Toll's contributions to QED were reported in a voluminous 1952 Princeton dissertation [4] that was never published, but nevertheless has strong connections to current research on the properties of the vacuum. These connections flow from the dissertation itself and from the efforts of Toll in training people and establishing organizations that could carry on the work. Some details about this little-known history are recounted in Appendix A.

One fundamental question that remains unresolved follows from the "Dyson Dilemma". Dyson showed [5] by a simple qualitative argument that an expansion of QED in a perturbative series possesses an essential singularity at the origin in the complex coupling-constant plane. This means that the unparalleled predictive power of QED that was just remarked upon comes from a theoretical foundation that has a basic weakness: a zero radius of convergence for perturbation theory. This is understood as the behavior of an "asymptotic series", where a finite number of terms in an expansion will give valid predictions, but at some higher order in the expansion the theory will fail. It is plain that this point has not been reached, but the Dyson Dilemma remains a worrisome indicator that contradictions may eventually arise.

<sup>a</sup> e-mail: reiss@american.edu

Toll suggested to the present author that an alternative procedure should be investigated to see if the Dyson problem persisted. The suggestion was to employ the methods of relativistic quantum mechanics (RQM) to solve a basic vacuum-structure problem exactly, and then to explore the analytical properties of the result obtained. The essential difference between RQM and QED for this task is that the electromagnetic field is regarded in RQM as a non-depletable external field that is not second-quantized. The problem selected for analysis was the creation of pairs by the collision of a single energetic photon with an intense beam of coherent photons. Within the context of RQM, the electron-positron pair could be treated by the exact Volkov solution [6–8] of the Dirac equation for a charged particle in a plane-wave electromagnetic field. The Ph.D. dissertation [9] based on this suggestion thus constituted the underlying theory for the eventual laboratory realization of this process by Burke et al. [10].

Section 2 that follows reveals the outcome of that work motivated by the Dyson Dilemma. RQM does not suffer from the singularity at the origin of QED, but the expansion parameter itself is altered from the fine-structure constant  $\alpha$  to an intensity-dependent object that retains a proportionality to  $\alpha$ . Further, it was shown that although the RQM theory has no essential singularity at the origin, there are other singularities in the complex coupling-constant plane that place intensity-dependent limits on the radius of convergence of a perturbative expansion in the background (i.e. laser) field. These limits on a perturbative expansion in the laser field are of considerable current importance in the application of strong laser fields to physical problems.

The properties of the experimental observation of photon-multiphoton pair production [10] are examined in Section 3. In particular, the experimentalists regarded their result to be perturbative in the laser field employed for the multiphoton background field, and they described the process as the scattering of light by light. Both of these assessments are questioned here. The experiments were nevertheless historic in that it was the first laboratory creation of particles with mass from an initial state possessing no mass whatever [11]. That is, although the production of pairs in the presence of the Coulomb field from a spectator particle is commonplace, and the annihilation of particle-antiparticle pairs into pure energy is equally well-known, nevertheless the creation of mass from an initial state possessing only electromagnetic energy was observed for the first time in 1997 [10] at the Stanford Linear Accelerator Center (SLAC). In the dissertation of the present author, and in the publications that followed from it [12,13], there is sufficient information to show definitively that the SLAC experiment was not perturbative, and the seeming perturbative behavior arising from the slope of a log-log plot of pairs produced as a function of field intensity, is a known artifact of strong-field physics [14]. As a separate matter, it is not clear to the present author how a process with no outgoing photon lines can be described as “photon-photon scattering”. A more accurate descrip-

tion is “absorption of light by light” [9,12] or, to be very explicit, “photon-multiphoton pair production”.

It is argued in Section 4 that, in physical processes arising from strong laser fields, the use of RQM methods in place of a fully second-quantized theory is entirely justified. First, the notion that there is a need to have an infinite sea of occupied negative energy states is contested. This assumption underlies the often-cited argument that the occupied negative-energy states require the use of second quantization of the electron and of the electromagnetic field. Then the utility of the Volkov Green’s function is established. Such a Green’s function was first presented by Schwinger [1], but it is of such complicated and implicit form that this author is unaware of any practical application of it. However, with the simple hypothesis of Floquet symmetry (i.e. the presence of a monochromatic field) it is possible to write a Green’s function with transparent properties [15]. Specifically, it is clear how the usual particle-antiparticle poles in a Feynman propagator give rise to families of generalized Floquet states associated with those poles.

The material in Section 4 contains also the explicit demonstration that a perturbative expansion in QED can, in sufficiently simple circumstances, be summed exactly to yield the same result as a nonperturbative Volkov-function theory of the same problem [16,17]. An interesting aspect of this work is the identification of those terms in an infinite-order perturbative expansion that give rise to the intense-field mass shift [9,12,13,18,19]. It also becomes clear why the mass shift does not arise in any finite order of perturbation theory.

Section 5 contains a brief discussion of the properties of the “Volkov electron” that distinguish it from an ordinary electron. These properties include the fact that a Volkov electron is not a simple spin-1/2 particle, but rather it exists as a particle in an intensity-dependent superposition of many angular momentum states.

Appendix A reviews some of the history of vacuum QED as it followed from the work of Wheeler, Feynman, and Toll.

Appendix B contains a demonstration, somewhat peripheral here but important in general, that the common practice in special relativity of viewing the electron mass as a velocity-dependent variable is to be deplored. It is not only unnecessary, but it needlessly destroys the covariance of any theory.

## 2 The Dyson dilemma

In a brief paper, Dyson [5] presented the argument that QED does not possess the analytical properties necessary for a convergent perturbation expansion. A convergence investigation requires the properties of the full theory as revealed in the complex coupling-constant plane. A simple physical argument reveals that a change in the sign of the coupling constant  $\alpha$  (i.e. its behavior on the negative real axis) leads to a divergent result. This property is sufficient to conclude that there must be an essential singularity at the origin in the complex  $\alpha$  plane, and thus

the radius of convergence is zero for an expansion of QED in powers of  $\alpha$ . The foundations of the remarkably useful and accurate Feynman-Dyson covariant perturbation theory are thus placed in question. To date, there has been no refutation of the Dyson argument. QED perturbation theory has, in consequence, been regarded as an asymptotic expansion that is well-behaved up to some order in the expansion, but must eventually diverge. The remarkable success of covariant perturbation theory, now known to match the precisely measured value of the anomalous magnetic moment of the electron to within one part in  $10^{12}$  [2,3], shows that perturbation theory does not suffer from its asymptotic character to eighth order.

## 2.1 RQM in place of QED

The question arises whether RQM has the same convergence problem as QED. A way to investigate this matter is to find and execute a calculation within RQM that avoids a perturbative expansion, and to then explore the result for its analytical properties. The suggestion of Toll was to employ Volkov solutions [6–8] to evaluate the production of electron pairs in the collision of an energetic photon with an intense beam of photons; a problem that can be described as the “absorption of light by light”. This proposal was made in 1955, before the advent of the laser. The energetic photon can be treated as a first-order perturbation, but the description of the “background” intense photon field was represented as the promotion of a Volkov electron from a negative energy state to a positive energy state, a standard procedure in “hole theory” to represent the production of an electron-positron pair. The expression “Volkov electron” has the meaning that the interaction of the electron with the field is expressed in terms of the known exact solution of the Dirac equation for an electron in a plane-wave electromagnetic field. The problem is thus exact in terms of the background field, within the RQM context.

It is often stated that the existence of negative energy states means that they must be filled, so that the Pauli Principle will serve to prevent the spontaneous decay of electrons into the negative-energy continuum. It requires an infinite number of electrons to accomplish that feat. That, in turn, then requires a many-body theory. Exactly that hole-theory language was used in the preceding paragraph in describing pair production as the promotion of an electron from a negative-energy state into a positive energy state. However, the perceived necessity of filling all negative energy states is misleading. Consider, for instance, the relativistic treatment of spinless particles. That can be done with the Klein-Gordon equation, and that equation also predicts negative energy states. One cannot evade the problem of spontaneous decay of positive energy bosons into negative energy states by recourse to the Pauli Principle, since the Pauli Principle applies only to fermions like the electron, and not to bosons like spin-zero particles. This apparent paradox with the Klein-Gordon equation prevented its practical use until it was pointed out by Pauli and Weisskopf [20] that the

negative energy states of a particle were really equivalent to positive energy states of the antiparticle. The seeming promotion of a particle from a negative energy state into a positive energy state is equivalent to the simultaneous production of a particle-antiparticle pair, both with positive energy.

The same procedure employed with the Klein-Gordon equation can be applied to the Dirac equation, thus removing the putative need for a filled sea of negative energy states along with its concomitant requirement for a many-body theory. That replacement of a negative energy particle by a positive energy antiparticle appears in the Feynman rules, where the production of a pair is represented by the conversion of an electron moving backward in time into an electron moving forward in time. The electron moving backward in time is equivalent to an electron in a negative energy state, and so the Feynman diagram representation of pair production involves only one continuous electron line.

A further assurance that it is not necessary to use a second-quantized theory in order to fill the sea of negative-energy fermions comes from the fact that the Feynman rules are identical for RQM and QED. The difference is that the diagrams that must be considered are specified in QED, whereas in RQM the necessary diagrams to be treated are decided from ad hoc considerations.

## 2.2 Expansion parameter in RQM

The first interesting thing to emerge from the investigation of convergence in RQM is that the fine structure constant never appears in isolation. It is always multiplied by an intensity-dependent factor such that the expansion in a perturbative series must be expressed as an expansion in the dimensionless quantity

$$z_f = 2U_p/mc^2, \quad (1)$$

where  $U_p$  is the ponderomotive energy, that is, the minimal interaction energy of a charged particle with the field. There are alternative ways to express equation (1). If the plane-wave field has the amplitude  $A_0$  for the vector potential as expressed in Gaussian units and in the radiation gauge, then

$$U_p = e^2 A_0^2 / 4mc^2, \quad (2)$$

$$z_f = e^2 A_0^2 / 2m^2 c^4, \quad (3)$$

where  $e$  is the electron charge. Another relation follows from the introduction of the photon density  $\rho$ , given by

$$\rho = A_0^2 \omega / 8\pi \hbar c^2, \quad (4)$$

$$z_f = \alpha 2\rho \lambda (\lambda_c / 2\pi)^2, \quad (5)$$

where  $\omega$  is the angular frequency of the field and  $\lambda_c$  is the electron Compton wavelength. Equation (5) has important physical content. It exhibits the manner in

which the fine structure constant enters into the intensity parameter  $z_f$ . It also shows that the number of background photons that enter into the process are all those photons contained in a volume with a transverse dimension given by  $\lambda_c$  and a length  $\lambda$ . One could guess this result on physical grounds. The effective size of a free electron for interaction with an electromagnetic field is given by the Compton wavelength, so that its cross section depends on the square of  $\lambda_c$ . A characteristic length of the field is its wavelength. Hence,  $\lambda\lambda_c^2$  appears as the effective volume in equation (5).

The parameter  $z_f$  serves as an expansion parameter, a general indicator of field intensity, and a direct measure of the “added mass” term that is discussed later in this paper.

### 2.3 Convergence-limiting singularities in RQM

Using a bounding procedure in the complex  $z_f$  plane, it is possible to show that the radius of convergence of a perturbation expansion in powers of  $z_f$  is, in general, nonzero [13]. However, singularities do exist, so that there is an upper limit on the radius of convergence. That upper limit is the intensity at which the first channel closing occurs. For low field intensity, if  $n_0$  photons is the smallest number necessary to satisfy 4-momentum conservation conditions, then increasing the field intensity will eventually lead to a point at which supplying an amount of energy necessary to provide the ponderomotive energy  $U_p$  to the electron requires at least  $n_0 + 1$  photons. An electron cannot be a physical particle unless it has the minimum energy  $U_p$  required to be a free charged particle in the field. The point at which the minimum number  $n_0$  must be replaced by  $n_0 + 1$  is called a channel closing, and it represents non-analytical behavior in the complex  $z_f$  plane.

In sum, RQM does not exhibit the Dyson Dilemma, but it does have an upper limit on its radius for the convergence of a perturbative expansion, and that limit is given by the first channel closing.

## 3 Exact equivalence of QED and RQM for laser-induced phenomena

In view of the equivalence of perturbative expansions in QED and RQM that follows from the identity of the Feynman rules in both cases, the question arises about how QED and RQM are related in fields that are nonperturbatively intense. This is a far more difficult question to ask than in the perturbative case, since there it is only necessary to examine individual diagrams for each order of perturbation theory, but a complete result must be explored in the nonperturbative case.

An opportunity presents itself for laser-induced phenomena, since one can then examine QED with the condition that one mode of the electromagnetic field is heavily populated and the other modes can be ignored or treated perturbatively. This is the premise adopted by Fried and

Eberly [16]. In view of the extreme analytical demands made by such a calculation, they selected the simplest possible case to explore. The problem analyzed was the Compton scattering of a zero-spin “electron” in a circularly polarized field. In a scalar electron context, there are diagrams in which a single photon is emitted or absorbed (arising from  $\mathbf{p} \cdot \mathbf{A}$  terms) and diagrams where two photons interact with the electron at the same vertex (arising from  $\mathbf{A}^2$ ). The advantage of using a circularly polarized field is that this eliminates all  $\mathbf{A}^2$  diagrams where two photons are absorbed or two are emitted. All that remains of the  $\mathbf{A}^2$  contribution are diagrams where one photon is absorbed and one emitted at the same vertex. It is further assumed that all electron self-energy diagrams make small contributions as compared to the direct diagrams that can be written. It can be confirmed [21] that this is an acceptable approximation even at fairly low photon densities.

The procedure employed in reference [16] was to write all possible Feynman diagrams meeting the constraints listed above, and then to find exact sums of the subclasses of diagrams that exist. An exact sum is essential, of course, if a nonperturbative result is to be obtained. It was found possible to express these sums in terms of continued fractions that could be evaluated in closed form. The end result was identical to that found in RQM using Volkov solutions [18,19,22] with one exception: the “mass-shift term” was not present.

In a strong laser field, the usual mass-shell expression is

$$p^2 = m^2 c^2, \quad (6)$$

where the relativistic conventions are those of Bjorken and Drell [23]. That is:  $p^2 \equiv p^\mu p_\mu$ , and a time-favoring real metric is employed such that

$$\begin{aligned} a^\mu b_\mu &= a_\mu b^\mu \\ &= a^0 b^0 - \mathbf{a} \cdot \mathbf{b}, \end{aligned} \quad (7)$$

where the letters in boldface are 3-vectors. In contrast to equation (6), all physical problems evaluated with Volkov solutions [9,12,18,19,22] show that a Volkov electron obeys the mass-shell condition

$$\begin{aligned} (p^\mu - nk^\mu)(p_\mu - nk_\mu) &= m^2 c^2 (1 + z_f) \\ &\equiv M^2 c^2. \end{aligned} \quad (8)$$

The quantity  $M$  is the dressed mass of the electron,  $k^\mu$  is the propagation 4-vector of the plane-wave field, and the quantity  $n$  is an integer.

Equation (8) can be interpreted as showing that the Volkov electron can exist in many different states, each corresponding to an ordinary electron carrying with it the full 4-momentum of  $n$  photons, and having a squared mass greater than the square of the normal electron mass by the factor  $(1 + z_f)$ .

The absence of the mass shift of equation (8) in reference [16] at first led to a controversy about the very existence of this phenomenon [24,25]. However, this confusion was settled in a convincing and informative way. In the course of the diagram summations in reference [16], a

class of diagrams was encountered in which the electron was exactly on the mass shell in an intermediate state. These diagrams thus had a zero in the denominator (i.e. in the inverse propagator factor), and such terms were discarded as being unphysical. However, those terms were re-examined in reference [17], and it was found that there was a finite result when all such nominally infinite terms were summed exactly. The outcome is exactly the added mass term  $m^2 c^2 z_f$  of equation (8).

As a corollary of the above result, it is possible to understand why the mass shift does not occur in any finite order of perturbation theory. The mass shift is thus a clear consequence of the failure of perturbation theory in very strong fields.

The mass alteration can be made very large with the next generation of lasers. However, to achieve a mass increase sufficient to make the field-dressed electron mass as great as that of a nucleon, a laser operating at a wavelength of 800 nm would have to have an intensity of about  $1.4 \times 10^{25}$  W/cm<sup>2</sup>.

#### 4 Volkov Green's function

The Green's function for a Volkov electron was first obtained by Schwinger [1]. However, the algebraic form of Schwinger's Green's function is so complicated that it is very difficult to extract any physical insight from it. A different form of the Volkov Green's function that did not have this difficulty was later found [15] with the addition of the hypothesis that a Greens function for a single mode of the electromagnetic field must have the periodicity of the field. This is sometimes called the Floquet property.

An appreciation for the properties of the Volkov electron vis-a-vis that of an ordinary electron can be obtained by comparing the two results for a simple example. The example is that of a "scalar" or spinless electron in a circularly polarized field. That is, the Klein-Gordon Green's function will be examined. (Ref. [15] also presents the spinor Green's function.)

The Green's function sought satisfies the equation

$$\left[ (i\partial^\mu - eA^\mu)^2 - m^2 \right] \mathcal{G}(x, x') = \delta(x, x'), \quad (9)$$

where the squared operator represents the combination of contravariant and covariant factors exemplified by the left-hand side of equation (8); here and hereafter,  $\hbar = c = 1$ ; and the right-hand side of equation (9) is the Dirac delta function. With no field interaction terms, equation (9) reduces to

$$\left[ (i\partial^\mu)^2 - m^2 \right] G(x, x') = \delta(x, x'). \quad (10)$$

A solution for equation (10) is

$$G(x, x') = \frac{1}{(2\pi)^4} \int d^4 p e^{-ip \cdot (x-x')} \frac{1}{p^2 - m^2 + i\epsilon}, \quad (11)$$

and the corresponding solution for equation (9) is

$$\mathcal{G}(x, x') = \frac{1}{(2\pi)^4} \int d^4 p e^{-ip \cdot (x-x')} \times \sum_{j=-\infty}^{\infty} \frac{\pi J_{-\sigma}(\zeta) J_{\sigma+j}(\zeta)}{2p \cdot k \sin \pi \sigma} e^{-ij(k \cdot x - \rho)}, \quad (12)$$

$$\sigma = \frac{(p^2 - M^2)}{2p \cdot k}, \quad (13)$$

$$\zeta e^{i\rho} = \frac{eA_0 p \cdot \varepsilon}{p \cdot k}, \quad (14)$$

$$\rho = \arctan \left( \frac{p \cdot \text{Im } \varepsilon}{p \cdot \text{Re } \varepsilon} \right), \quad (15)$$

$$A^\mu = A_0 \text{Re} (\varepsilon^\mu e^{ik \cdot x}). \quad (16)$$

Equations (11) and (12) appear to be so different that it is difficult to see physically what these differences mean. However, the situation is greatly clarified if the  $dp^0$  part of the  $d^4 p$  integration is carried out.

For equation (11), the well-known result is

$$G(x, x') = -\frac{i}{(2\pi)^3} \int d^3 p \frac{\exp [i\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}') \mp iE(x^0 - x'^0)]}{2E}, \quad (17)$$

$$E = (\mathbf{p}^2 + m^2)^{1/2}. \quad (18)$$

The upper choice of the ambiguous  $\mp$  sign in equation (17) pertains to  $(x^0 - x'^0) > 0$ , while the lower choice holds if  $(x^0 - x'^0) < 0$ . This result follows from the poles that exist in the denominator in equation (11), which can be rewritten

$$p^2 - m^2 + i\epsilon = (p^0)^2 - \mathbf{p}^2 - m^2 + i\epsilon = 0, \quad (19)$$

$$p^0 = \pm (E^2 - i\epsilon)^{1/2}, \quad (20)$$

with  $E$  as defined in equation (18). As written in equation (20), a value of the infinitesimal quantity  $\epsilon$  that is positive always leads to the particle lying below the real axis and the antiparticle pole lying above the real axis. This is consistent with a Feynman propagator where a completion of the contour on an arc in the lower part of the complex  $p^0$  plane for  $(x^0 - x'^0) > 0$  will enclose the particle pole, and a completion of the arc in the upper complex  $p^0$  plane for  $(x^0 - x'^0) < 0$  will enclose the antiparticle pole. That is, positive-energy electrons are propagated forward in time and negative-energy electrons are propagated backward in time, in accordance with the Feynman prescription.

There will be poles in the denominator of equation (12) whenever  $\sigma = n = \text{integer}$ . Equation (13) yields

$$p^2 - M^2 = 2np \cdot k. \quad (21)$$

Since the propagation vector  $k^\mu$  for the field is on the light cone, meaning that  $k^2 = k^\mu k_\mu = 0$ , then equation (21) factors into

$$(p^\mu - nk^\mu)(p_\mu - nk_\mu) = M^2, \quad (22)$$

which is precisely equation (8). Introducing the notation

$$\mathcal{E} = (\mathbf{p}^2 + M^2)^{1/2}$$

in analogy to equation (18), the poles of the integrand in equation (12) are at

$$p^0 = n\omega \pm \left[ \mathcal{E}^2 - 2n\omega p^3 + (n\omega)^2 - i\epsilon \right]^{1/2}. \quad (23)$$

In equation (23), the occurrence of the component  $p^3$  of the 3-momentum means that a spatial coordinate system has been selected such that the laser field propagates in the  $x^3$  direction. For further comments about  $\mathcal{G}(x, x')$  in various limiting cases, see the original article [15].

The correspondence between equations (20) and (23) is clear. The electron and positron poles following from equation (20) are now replaced by the same poles with the mass shift incorporated and, in addition, there are families of sideband poles associated with each value of  $n$  and dependent on whether  $n$  is positive or negative. It is also true that the infinitesimal complex-plane displacement  $\epsilon > 0$  will cause all positive-energy Volkov electrons to propagate forward in time and the negative-energy Volkov electrons to propagate backward in time.

## 5 Photon-multiphoton pair production at SLAC

The experiment [10] at SLAC in 1997 was historic. It was the first experiment to produce particles with mass from an initial state that had no mass at all. One might say that it was a direct demonstration of the polarization of the vacuum, with the final particles on the mass shell (as opposed to being virtual). It was also an explicitly non-perturbative photoprocess in the elementary particle domain, in contrast to earlier experiments on nonperturbative atomic ionization. It represented a realization of the theoretical calculation published [12] 35 years in advance of the experiment. (There is an irony here. The paper [12] was initially sent to the Physical Review for publication, but it was rejected on the grounds that the required field intensity did not exist and never would exist. It was sent to the Journal of Mathematical Physics on the recommendation of the referee, who regarded it as nothing more than a mathematical exercise. It took only 35 years to prove him wrong.)

There are, however, several aspects of the reporting of the SLAC work that would bear reconsideration. It was already mentioned in the Introduction above that the title of the paper: ‘‘Positron Production in Multiphoton Light-by-Light Scattering’’, is a misnomer. The scattering of light by light requires outgoing scattered photons that were not part of the SLAC experiment. The scattering of light by light is a process that is of significantly lower probability than photon-multiphoton pair production, and it has not been observed to date.

A different matter that is of central importance is that Burke et al. [10] regard their experiment as being a high

order (fifth order) perturbative process. They reach this conclusion on the basis of a yield-vs.-intensity plot that seems to show a clear dependence on intensity  $I$  as  $I^5$ . It is shown here that the process was well beyond the radius of convergence of a perturbation expansion, and that the  $I^5$  dependence is an artifact of a type already known in strong-field ionization of atoms.

The basic kinematics will be established first. The laboratory environment has the two types of photons colliding head-on. If the frequency of the energetic photon is  $\tilde{\omega}$  and that of a laser photon is  $\omega$ , then the scalar 4-product of the propagation 4-vectors is a Lorentz invariant whose value is

$$k^\mu \tilde{k}_\mu = \omega \tilde{\omega} - \mathbf{k} \cdot \tilde{\mathbf{k}} = 2\omega \tilde{\omega}. \quad (24)$$

This means that, if  $n$  photons from the laser participate in a given process, the quantity  $n\omega \tilde{\omega}$  is an invariant. In particular, if a Lorentz transformation is performed along the common direction of propagation, it is possible to transform to the center-of-momentum frame. The threshold condition in that frame is that

$$n\omega' \tilde{\omega}' \geq m^2, \quad (25)$$

since the electron pair will be at rest at threshold. The primes on the photon frequencies refers to the fact that they have been Lorentz transformed from the laboratory frame to the frame where the momenta balance, so that

$$n\omega' = \tilde{\omega}' \geq m, \quad (26)$$

where the last inequality in equation (26) comes directly from (25). Because  $n$  is a fixed integer, then  $n\omega \tilde{\omega}$  is a Lorentz invariant whose value is given in equation (25). It is known [10] in the laboratory frame that  $\omega = 2.35$  eV (because  $\lambda = 527$  nm) and  $\tilde{\omega} = 29.2$  GeV. Therefore the smallest possible  $n$ , labeled  $n_0$  is

$$n_0 \geq \frac{m^2}{\omega \tilde{\omega}} = \frac{(5.11 \times 10^5 \text{ eV})^2}{(2.35 \text{ eV})(2.92 \times 10^{10} \text{ eV})} = 3.805, \quad (27)$$

$$n_0 = 4. \quad (28)$$

Equation (28) follows from (27) in that  $n_0$  is the smallest integer greater than 3.805; that is,  $n_0 = 4$ .

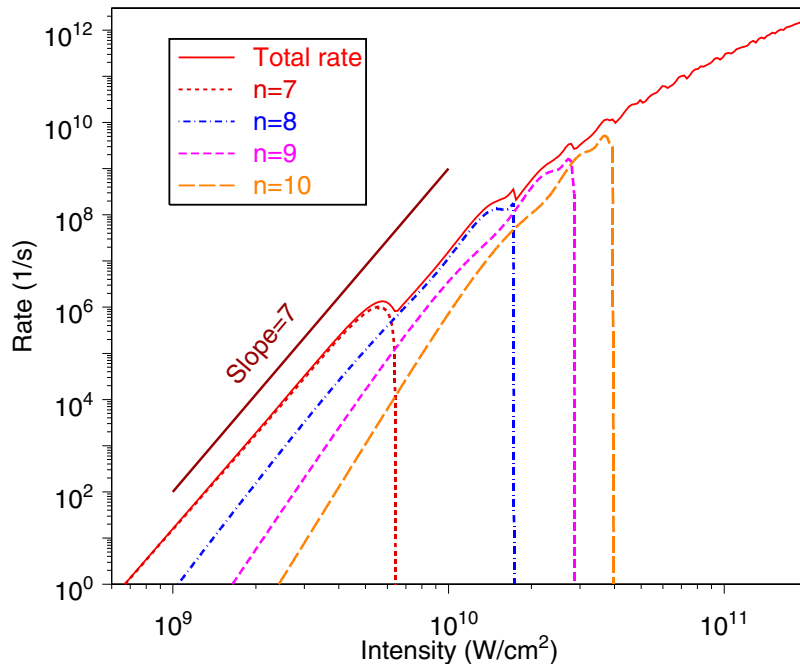
However, the results (27) and (28) do not account for the fact that the electron and positron are created in the presence of the laser field, so that the rest energy of each particle is not  $m$ , but rather it is the  $M$  of equation (8). The peak laser intensity given in reference [10] is  $1.3 \times 10^{18}$  W/cm<sup>2</sup>. From equations (1) or (3) or (5) one finds that  $z_f = 0.132$  in the SLAC experiment. Equation (8) then specifies that

$$M = m\sqrt{1 + z_f} = 5.44 \times 10^5 \text{ eV}. \quad (29)$$

The revised equation (27) then yields

$$n_0 \geq \frac{M^2}{\omega \tilde{\omega}} = 4.31, \quad (30)$$

$$n_0 = 5. \quad (31)$$



**Fig. 1.** (Color online) This figure is meant to illustrate the manner in which a nonperturbative laser environment can lead to a yield curve that appears to be perturbative. The specific example is the photodetachment of a hydrogen negative ion by a laser operating at  $10.6 \mu\text{m}$ . A uniform intensity is assumed, so that there is no “focal averaging”. A realistic laser pulse possesses a distribution of intensities that would smooth all of the irregularities seen in this figure. The lowest order perturbative process that could occur is  $n_0 = 7$ . To guide the eye, the figure shows a straight line with a slope of 7 on this log-log plot. The lowest order process begins with this slope, but starts to deviate towards a lesser slope shortly before closing altogether. This property is much more in evidence with  $n = 8$ , where the initial slope of 8 becomes a slope of 7 before the channel closes completely. Higher order channels also show this property of having an initial slope equal to the order of the particular channel, which then reduces to a lesser slope as lower-order channels close. The overall perception is of a slope that remains appropriate to the perturbative process, even as channels are closing, and perturbation theory is no longer applicable.

A comparison of equation (31) with (28) shows that the field intensity has caused a channel to close. It is then unequivocal from reference [13] that perturbation theory has failed. *The intensity is beyond the radius of convergence of perturbation theory.*

The upper limit on the intensity where the experiment could still be perturbative corresponds to the intensity at which  $M^2/\omega\tilde{\omega} = 4$ . Equation (29) then yields the value of  $z_f$  at which this occurs. This channel-closing threshold value of  $z_f^{cc}$  is

$$z_f^{cc} = \frac{4\omega\tilde{\omega}}{m^2} - 1 = 0.051. \quad (32)$$

In the terminology of Burke et al., the quantity they call  $\eta$  is exactly the same as  $z_f$  here. Figure 4 in reference [10] shows that the threshold for pair production occurs when  $z_f = 0.2$ . All the non-trivial yield data were found at higher intensities than this, and therefore all the data were accrued in the nonperturbative domain.

Burke et al. [10] show a yield graph that seems to indicate that the yield increases with the field intensity as  $I^5$ . This is consistent with equation (31), but the implication gathered by Burke et al. is that the linearity of a log plot of yield vs. intensity is evidence of perturbative behavior. This was a subject of a controversy in the 1980s on

the topic of atomic ionization, where a constant slope of a yield curve as a function of intensity on a log-log graph was also taken to signify perturbative behavior by a part of the community. However, it had already been shown in the original strong-field-approximation (SFA) paper [14] that this constant slope is deceptive. Figures 7 and 8 of that paper show clearly that, as channels close because of an increase in intensity, the sum of higher order channels conspire to make the slope appear constant over an extended domain of intensity beyond where the lowest-order channel has closed.

Figure 1 here is a recalculation of Figure 8 of reference [14] done with much better computer and graphics capabilities than existed in 1980; and with a more accurate analytical Hartree-Fock wave function for the ground state. The lowest-order slope of  $n_0 = 7$  for that problem was maintained for nearly a decade of intensity beyond the point at which perturbation theory has failed. It is instructive to observe the behavior of orders higher than  $n_0$ . For example, the  $n = 8$  curve begins with a slope of 8 at low intensity, but by the intensity at which the  $n = 7$  channel closes, the  $n = 8$  curve has decreased its slope from 8 to 7. In like fashion, the  $n = 9$  curve starts with a slope of 9, but that slope has diminished to about 7 at the intensity where the  $n = 9$  channel has closed.

Beyond the closing of the  $n = 9$  channel, the summed rate has already begun to fall to less than the slope of 7. The declining importance of the lower orders with replacement by a combination of higher orders is a form of “above-threshold ionization” (ATI) that is not manifested by separated peaks in the spectrum representing individual higher-order contributions, as in the early ATI work [14,26–28].

The conclusion is that the SLAC experiments were nonperturbative, and exhibited the above-described mimicking of a perturbative rate of increase of yield with intensity.

## 6 Volkov electron

In the SLAC experiments, as well as in the theory of relativistic atomic ionization [29], the general properties of a Volkov electron were essential, but details of its properties were submerged in the complexities of the specific experiments and calculations. There is, however, one aspect of a Volkov electron that merits special attention. That aspect is the fact that a Volkov electron can no longer be regarded as a spin-1/2 particle, although, of course, it remains a fermion.

To appreciate these remarks, the Volkov solution is presented here for the case of an electron in a monochromatic linearly polarized field:

$$\begin{aligned} \Psi^{Volk} &= \left(\frac{m}{EV}\right)^{1/2} \left(1 + \frac{e(k \cdot \gamma)(A \cdot \gamma)}{2p \cdot k}\right) u \\ &\times \exp \left[ -i \left( p \cdot x + \eta k \cdot x + \zeta \sin k \cdot x + \frac{1}{2} \eta \sin 2k \cdot x \right) \right], \end{aligned} \quad (33)$$

where the dot products are scalar products of 4-vectors,  $V$  is the volume for a box normalization of this free-particle wave function, the 4-vector  $\gamma^\mu$  has the four Dirac matrices as its components,  $u$  is a 4-spinor satisfying the condition

$$(p \cdot \gamma - m) u = 0, \quad (34)$$

$p^\mu$  is the energy-momentum 4-vector for the field-free electron, the quantity  $E$  in the normalization factor in equation (33) is just  $p^0$ , and the all-important coefficients in the exponential function in equation (33) are

$$\eta \equiv \frac{e^2 A_0^2}{4p \cdot k}, \quad (35)$$

$$\zeta \equiv \frac{e A_0 \mathbf{p} \cdot \boldsymbol{\epsilon}}{p \cdot k}. \quad (36)$$

In the interest of brevity, only the  $\eta$  factor defined by equation (35) will be examined.

To assess the magnitude of  $\eta$  it will be convenient to extract factors to produce the dimensionless 4-vectors  $\hat{p}^\mu = p^\mu/m$  and  $\hat{k}^\mu = k^\mu/\omega$ . Then  $\eta$  becomes

$$\eta = \frac{e^2 A_0^2}{4m\omega \hat{p} \cdot \hat{k}} = \frac{m}{2\omega} \frac{z_f}{\hat{p} \cdot \hat{k}}, \quad (37)$$

using the form (3) for  $z_f$ . If the quantity  $\hat{p} \cdot \hat{k}$  is regarded as near unity, then the magnitude of  $\eta$  is determined by  $(m/\omega)z_f$ . If  $\omega$  refers to a laser-generated field, then this factor can be extremely large. The exponential in equation (33) contains the quantity  $\eta k \cdot x$ . This can now be estimated to be

$$\eta k \cdot x = \eta(\omega t - \mathbf{k} \cdot \mathbf{r}) = \eta\omega \left( t - \hat{\mathbf{k}} \cdot \mathbf{r} \right) \approx \frac{m}{2} z_f \left( t - \hat{\mathbf{k}} \cdot \mathbf{r} \right). \quad (38)$$

If  $|\mathbf{r}|$  is of the order of the size of a nucleus, the usual assumption is that a laser field can certainly be viewed as something that can be treated by a dipole-approximation theory. That is, the usual expectation is that

$$\omega t - \mathbf{k} \cdot \mathbf{r} \approx \omega t \quad (39)$$

for a nucleus in a laser field. However, in the combination that appears in equation (38), it is not at all clear that the dipole approximation is valid. If the nuclear size is represented by  $R$ , then

$$\eta \mathbf{k} \cdot \mathbf{r} = O\left(\frac{m}{2} z_f R\right) = O\left(z_f \frac{R}{\lambda_c}\right), \quad (40)$$

where  $\lambda_c$  is the electron Compton wavelength that appeared in equation (5) above. The result in equation (40) need not be negligibly small, and so the dependence on the position vector  $\mathbf{r}$ , even when limited by a nuclear radius, can still introduce important orbital angular momentum components into the interaction of a Volkov electron with a nucleus. It can also be shown readily that the  $\eta \sin 2k \cdot x$  and  $\zeta \sin k \cdot x$  terms in the exponential of equation (33) can make important contributions.

## Appendix A: Brief history

The 1965 Nobel Prize in Physics was awarded jointly to Feynman, Schwinger, and Tomonaga for their work in QED. This is obviously well-known, as is also the work of Dyson in showing the equivalence of the three methods. Less known is the role of Wheeler as, in a sense, the Godfather of much of the work on QED fundamentals that was going on in the 1940s and early 1950s. Feynman was a student of Wheeler, obtaining his degree in 1942. Wheeler and Feynman were still writing joint papers up to 1949 [30,31]. One of Wheeler’s most brilliant students was Toll, whose 1952 Ph.D. dissertation on the fundamentals of vacuum QED is famous within a limited group of people, but was never published. However, almost completely unknown is the existence of a group of theoreticians with a focus on nonperturbative, strong-field physics that was established with the help of Toll in about 1961, thus predating the voluminous work of this nature that was performed at the Lebedev Institute in Moscow. A brief account of this history will be given here.

During World War II, many US Government laboratories gathered very talented groups of scientists, many of whom worked on fundamental problems in physics with no obvious relationship to the basic mission of the host laboratory. The concept was that people of superior talent could not be retained without permitting them to work on topics of their own choice, while still being available to aid in the solution of demanding laboratory projects when necessary. One such laboratory was the US Naval Ordnance Laboratory (NOL) in White Oak, Maryland, located not far from the University of Maryland. Among their illustrious alumni were Bardeen of semiconductor fame, and Misner, later a collaborator with Wheeler and Thorne on the classic general relativity textbook “Gravitation”.

In the 1950s, NOL decided to form a Nuclear Physics Division, and they requested the assistance of Toll. Toll had moved from his Princeton Ph.D. in 1952 to Chairman of the Physics Department at the University of Maryland in 1953. The present author (HRR) was a student of Toll at the University of Maryland, working on the dissertation here listed as reference [9]. In 1959 (until 1969), HRR became head of the Nuclear Physics Division. This had the advantage of putting him in contact with two fascinating individuals who visited NOL and the Nuclear Physics Division on a regular basis. One was Clyde L. Cowan, who was codiscoverer of the neutrino with Reines. Although Cowan died in 1974, the 1995 Nobel Prize in Physics was awarded to “Cowan and Reines” for the discovery of the neutrino. Cowan became a good friend as well as a collaborator. A patent was issued in 1970 on the neutrino detection of underground nuclear weapons tests, in the names of Cowan, HRR, and two other members of the Nuclear Physics Division. The other exceptional individual with whom HRR had extensive interaction was Joseph Weber, inventor of the maser and the pioneering developer of instrumentation for the detection of gravity waves [32].

The purpose of the above paragraphs is to introduce the Nuclear Physics Division at NOL, because it was there that the first research group was formed for the study of basic strong-field problems in electrodynamics. There were many activities in the Division including, of course, nuclear physics; but that portion of the Division that focused on the foundations of relativistic strong-field physics had the greatest long-term impact. The first products were references [12] and [13], but other members of the group started making major contributions. Many people participated, but only three will be mentioned here. A 1960 arrival was William M. Frank, a brilliant mathematical physicist whose life was tragically cut short by an aggressive case of multiple sclerosis. In 1961, Zoltan Fried joined the group. His major contributions have already been discussed in Section 3. The third essential member of the group was Joseph H. Eberly, who joined as a National Academy of Sciences Postdoctoral Fellow in the years 1963–1967, before he went to the University of Rochester to pursue a productive and influential career that continues to the present.

The strong-field physics group within the NOL Nuclear Physics Division was not only the first such group in existence, but its activities were seminal for much later research in this now-large and still-growing field. Nikishov and Ritus were the first of the Soviet physicists to publish their strong-field work [18], and reference [1] in their paper is reference [12] here. That citation has been largely overlooked, but there is a story attached to it that has been recounted personally to the present author by Nikishov and Ritus directly, and also by Nikolay Narozhny [33]. The Soviet Academician and Nobel Laureate V.L. Ginzburg saw reference [12], and was favorably impressed. He brought it to the attention of the Lebedev Institute physicists with the recommendation that the subject of nonperturbative strong fields, and the Volkov techniques employed in the paper had considerable promise, and was worth pursuing. Nikishov, Ritus, and Narozhny were unanimous and generous in crediting reference [12] with having “started strong-field physics”. There is more. Many people in the West were initially unaware of the work at the Lebedev Institute as well as of the earlier work of HRR. To them, the field was started by Brown and Kibble [19] in 1964. However, Brown and Kibble are quite clear in their paper that it was Zoltan Fried who called their attention to nonperturbative electrodynamics, and who alerted them to the usefulness of Volkov methods. In turn, Zoltan Fried became aware of these matters after he joined the Nuclear Physics Division at NOL. Therefore, it was this strong-field group at NOL that planted the seeds of this field of study in both the East and the West.

## Appendix B: Variable mass vs. covariance

Many researchers in the application of relativistic QED to practical problems accept the concept that the mass of a particle is velocity-dependent, depending on the velocity  $v$  through the relation

$$m = m_0\gamma, \quad (\text{B.1})$$

$$\gamma = (1 - v^2)^{-1/2}, \quad (\text{B.2})$$

where  $m_0$  is referred to as the “rest mass” of the particle, and the convention of setting  $c = 1$  is continued. This practice of treating the mass as variable is decried here on the grounds that it is both unnecessary and the cause of a needless loss of covariance.

A dependence on the concept of covariance is an essential part of the major progress in relativistic quantum field theory that followed from the work of Feynman, Schwinger, and Tomonaga. “Covariance” means that all quantities that enter into a theory should have well-defined properties under Lorentz transformations. For present purposes, only the Lorentz scalar and Lorentz vector need to be considered. A Lorentz scalar is simply a quantity that is the same in all inertial frames of reference. A Lorentz vector is defined to be a quantity that has the same transformation properties as the space-time

vector

$$x^\mu : (t, \mathbf{r}). \quad (\text{B.3})$$

A covariant velocity can be defined as the differential of  $x^\mu$  with respect to the proper time  $\tau$ ; a scalar quantity. That is, the covariant velocity is

$$u^\mu = \frac{d}{d\tau} x^\mu. \quad (\text{B.4})$$

It is then a simple matter to define a covariant momentum as the product of a Lorentz-scalar mass and a Lorentz-vector velocity:

$$p^\mu = m_0 u^\mu. \quad (\text{B.5})$$

So far, all is straightforward. Trouble arises when an increment in the proper time  $\Delta\tau$  in equation (B.4) is related to an increment in the laboratory time  $\Delta t$ , which experiences a Lorentz dilation with respect to  $\tau$ . The connection is

$$\Delta t = \gamma \Delta\tau, \quad (\text{B.6})$$

so that, in equation (B.4) one has

$$u^\mu = \gamma \frac{\partial}{\partial t} x^\mu = \gamma v^\mu, \quad (\text{B.7})$$

with the last expression serving as a definition of the local 4-velocity  $v^\mu$ . Equation (B.5) can then be expressed as

$$p^\mu = m_0 \gamma v^\mu. \quad (\text{B.8})$$

The mischief occurs when the factors in equation (B.8) are grouped as

$$p^\mu = (m_0 \gamma) v^\mu, \quad (\text{B.9})$$

and  $m_0 \gamma$  is viewed as a variable mass, as remarked in equation (B.1).

This introduction of a variable mass has taken the expression of the 4-vector momentum  $p^\mu$  as the product of a Lorentz scalar  $m_0$  and a Lorentz vector  $u^\mu$  in equation (B.5), to a new form (B.9) in which neither factor has any specific Lorentz character. This is both needless and harmful.

When written in the covariant form of equation (B.5), the subscript on the mass is unnecessary since there is only one mass, and that is the rest mass. Equation (B.5) can then be rewritten as

$$p^\mu = m u^\mu \quad (\text{B.10})$$

without any ambiguity.

## References

1. J. Schwinger, Phys. Rev. **82**, 664 (1951)
2. B. Odom, D. Hanneke, B. D'Urso, G. Gabrielse, Phys. Rev. Lett. **97**, 030801 (2006)
3. G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, B. Odom, Phys. Rev. Lett. **97**, 030802 (2006)
4. J.S. Toll, Ph.D. Dissertation, Princeton (1952), unpublished
5. F.J. Dyson, Phys. Rev. **85**, 631 (1952)
6. D.M. Volkov, Z. Phys. **94**, 250 (1935)
7. J.H. Taub, Ann. Math. **40**, 937 (1939); J.H. Taub, Rev. Mod. Phys. **21**, 388 (1949)
8. N.D. Sengupta, Bull. Calcutta Math. Soc. **39**, 147 (1947)
9. H.R. Reiss, Ph.D. Dissertation, Univ. of Maryland (1958)
10. D.L. Burke et al., Phys. Rev. Lett. **79**, 1626 (1997)
11. H.R. Reiss, Phys. Rev. Lett. **26**, 1072 (1971)
12. H.R. Reiss, J. Math. Phys. **3**, 59 (1962)
13. H.R. Reiss, J. Math. Phys. **3**, 387 (1962)
14. H.R. Reiss, Phys. Rev. A **22**, 1786 (1980)
15. H.R. Reiss, J.H. Eberly, Phys. Rev. **151**, 1058 (1966)
16. Z. Fried, J.H. Eberly, Phys. Rev. **136**, B871 (1964)
17. J.H. Eberly, H.R. Reiss, Phys. Rev. **145**, 1035 (1966)
18. A.I. Nikishov, V.I. Ritus, Z. Eksp. Teor. Fiz. **46**, 776 (1964), Sov. Phys. JETP **19**, 529 (1964)
19. L.S. Brown, T.W.B. Kibble, Phys. Rev. **133**, A705 (1964)
20. W. Pauli, V.F. Weisskopf, Helv. Phys. Acta **7**, 709 (1934)
21. J.H. Eberly, Phys. Rev. Lett. **19**, 284 (1965)
22. I.I. Goldman, Phys. Lett. **8**, 103 (1964)
23. J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964)
24. T.W.B. Kibble, Phys. Rev. **138**, B740 (1965)
25. Z. Fried, A. Baker, D. Korff, Phys. Rev. **151**, 1040 (1966)
26. P. Agostini, F. Fabre, G. Mainfray, G. Petite, N.K. Rahman, Phys. Rev. Lett. **42**, 1127 (1979)
27. P.H. Bucksbaum, M. Bashkansky, R.R. Freeman, T.J. McIlrath, L.F. DiMauro, Phys. Rev. Lett. **56**, 2590 (1986)
28. H.R. Reiss, J. Phys. B **20**, L79 (1987)
29. H.R. Reiss, J. Opt. Soc. Am. B **7**, 574 (1990)
30. J.A. Wheeler, R.P. Feynman, Rev. Mod. Phys. **17**, 157 (1945)
31. J.A. Wheeler, R.P. Feynman, Rev. Mod. Phys. **21**, 425 (1949)
32. J. Weber, *General Relativity and Gravity Waves* (Interscience, New York, 1961)
33. N.B. Narozhny, private communication